

Informed and augmented learning for the  
efficient simulation of complex systems

Francisco (Paco) CHINESTA

# Agenda

1. Contexte
2. Méthodes
3. Applications

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1. Contexte
2. Méthodes
3. Applications

## 21st century engineering:

- From design to operation
- From component to system of systems

### The Twin Continuum

#### VIRTUAL

- 100% Physics
- $U=U(F)$
- Software
- Processing time
- Resources
- Complex systems
- Variability
- Uncertainty

#### INFORMED

$u(x)$  AI-based  
such that  $L(u)=G$

- Less data
- Faster
- Explainable

#### AUGMENTED

##### - HYBRID

$$u(x)=u^P(x)+u^D(x)$$

#### DIGITAL

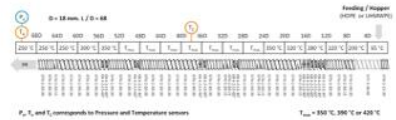
- Unknown or poor model
- $U_i, F_i, i=1,2, \dots$
- Machine learning:  $U(F)$
- Data cost
- Which data, where & when?
- Extrapolation
- Explanation

# Agenda

1. Contexte
2. **Méthodes**
3. Applications

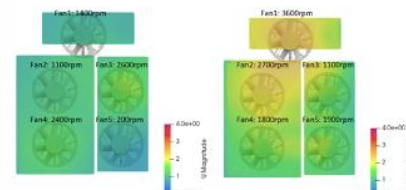
# I – Data Typology

## Lists

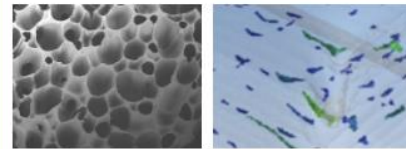


## Images

FEM

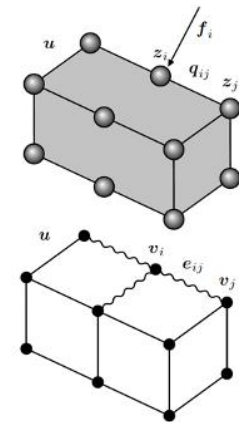


EXP



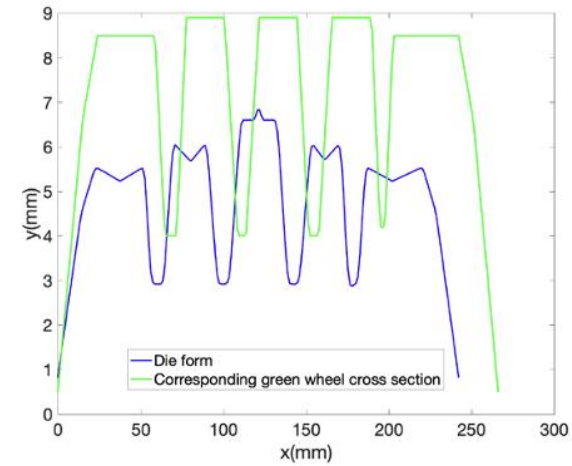
Descriptors, metrics  
& data transformation

## Graphe

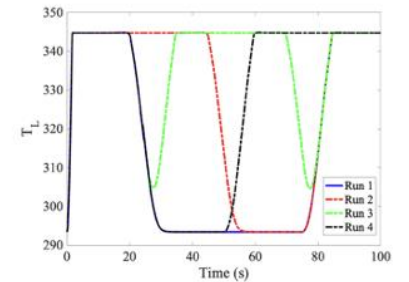


Connectivity,  
balance & behavior

## Curves



## Time series

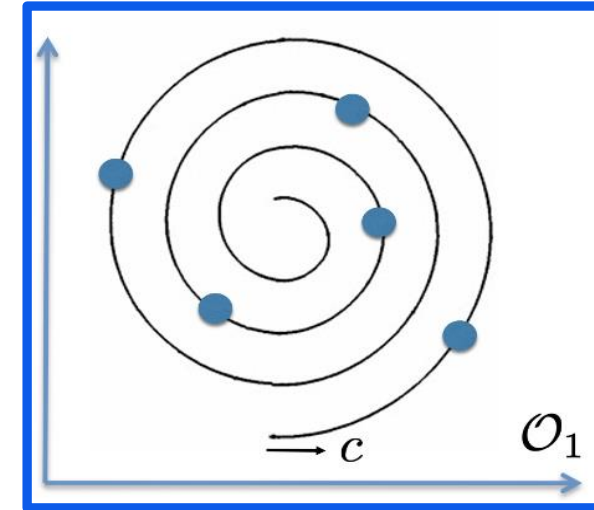
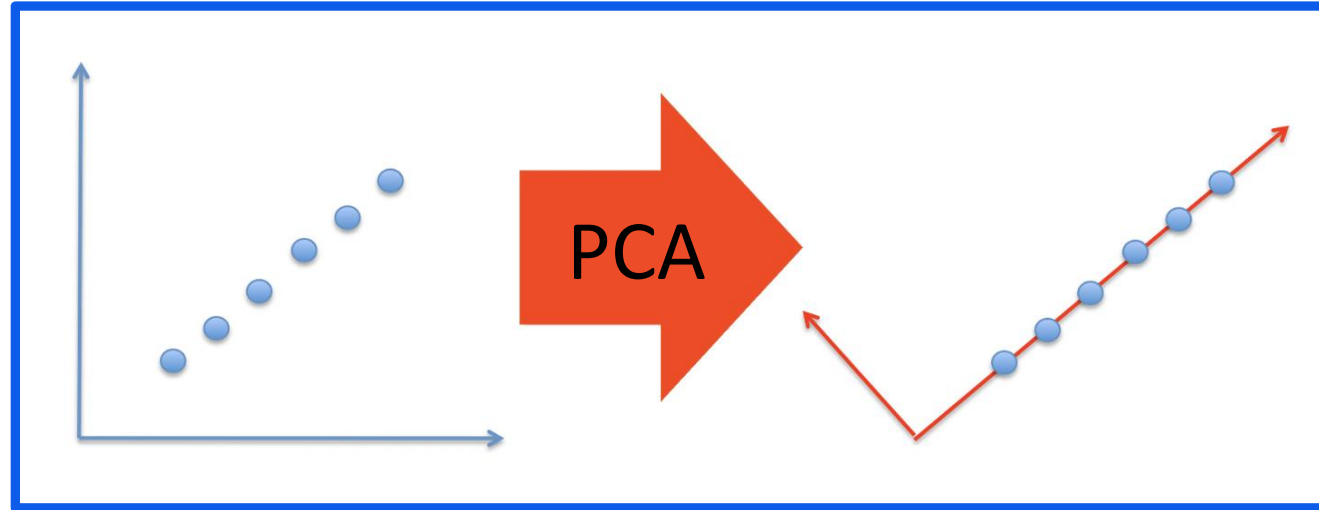


Causality, stability  
description and metrics

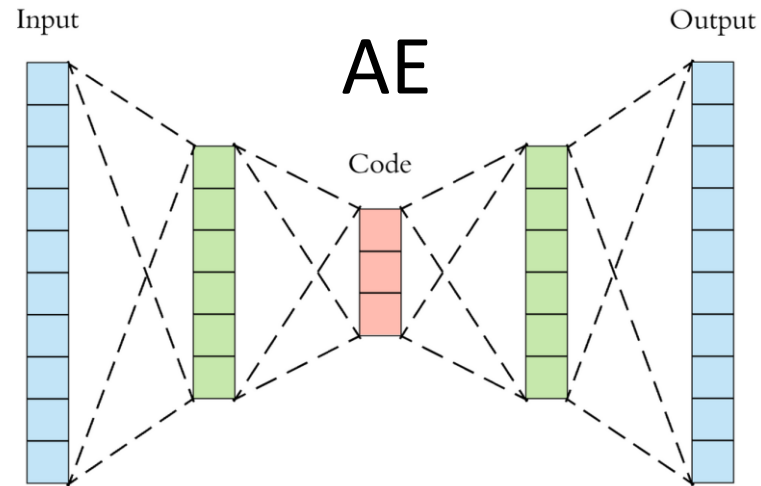
- ☐ Volume
- ☐ Useful versus useless
- ☐ Completeness

# I – Data Reduction

LINEAR



NONLINEAR



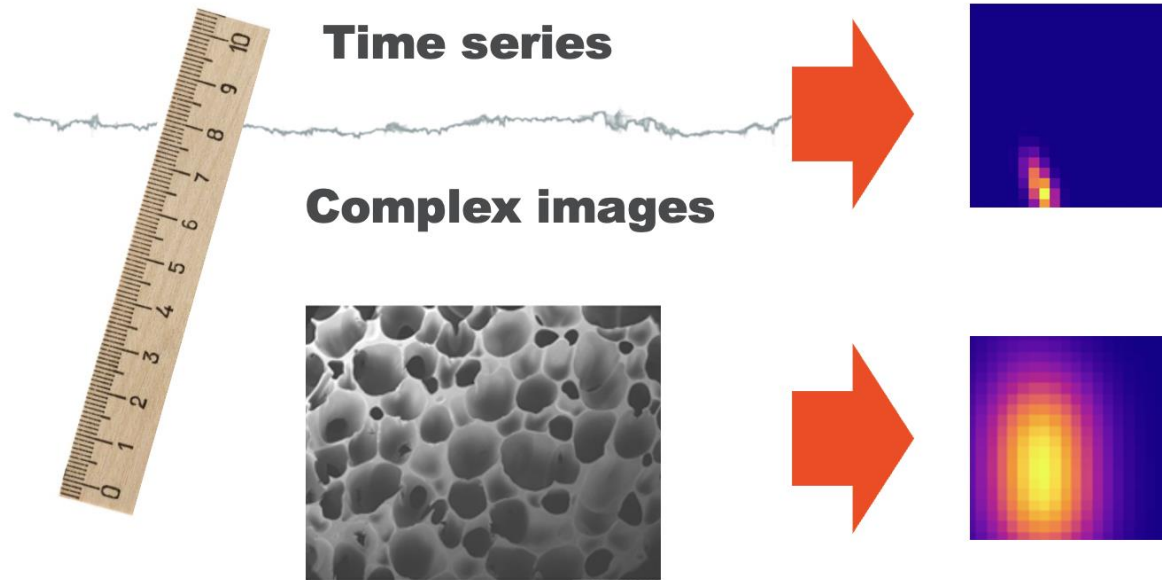
Manifold Learning

$$\mathcal{L}(\mathbf{y}, \mathbf{y}') = \|\mathbf{y} - \mathbf{y}'\|^2 = \|\mathbf{y} - \sigma'(\mathbf{W}'(\sigma(\mathbf{W}\mathbf{y} + \mathbf{b})) + \mathbf{b}')\|^2$$

# I – Data: Metrics

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## Topological Data Analysis - TDA

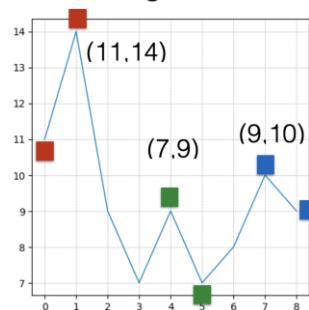


**A sort of goal oriented  
QR code / Passport**

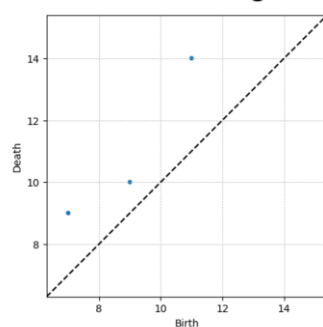


**BUT in a vector space**

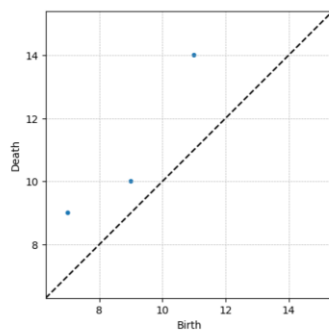
Pairing min-max



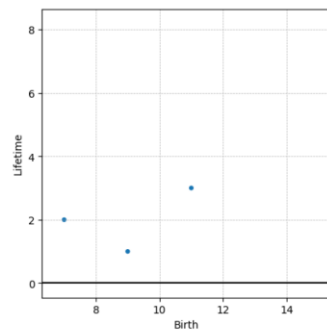
Persistence diagram



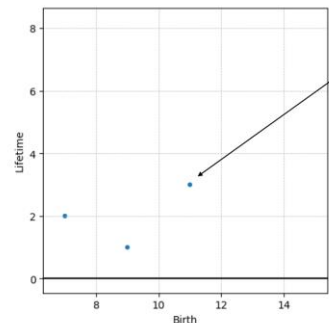
Persistence diagram



Lifetime diagram

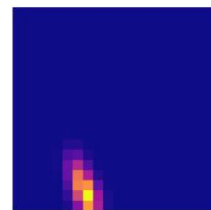


Lifetime diagram

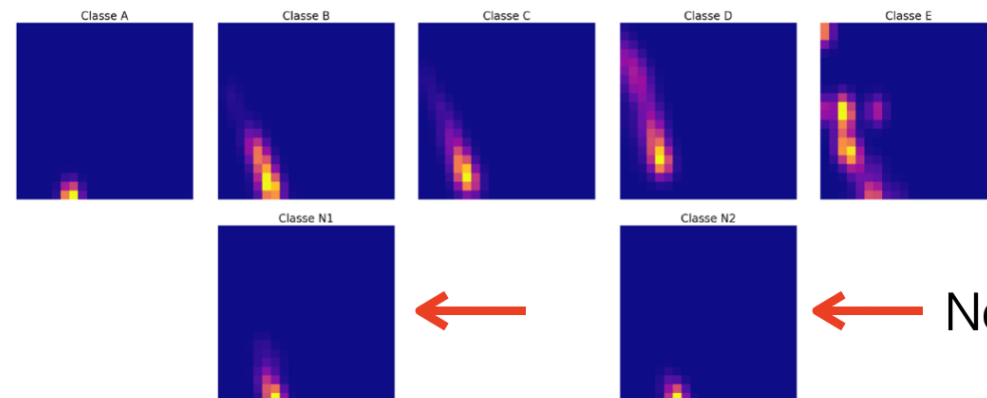
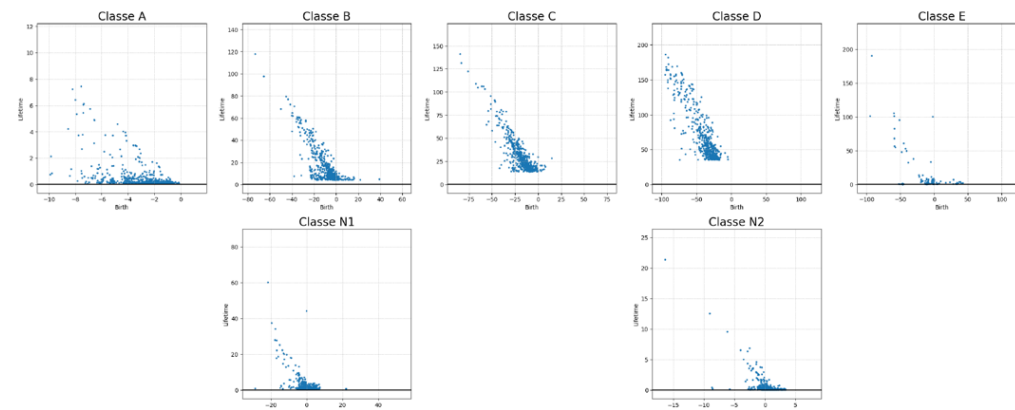
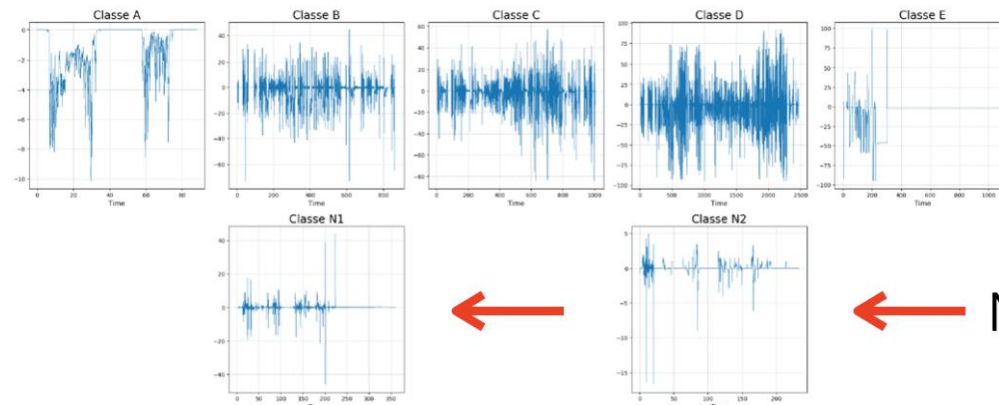


$$\rho_S(u, v) = \sum_{(x, y) \in T(S)} w(x, y) g_{(x, y)}(u, v).$$

$$\mathcal{PI}_{P_i}(S) = \iint_{P_i} \rho_S(u, v) du dv.$$



Persistence image

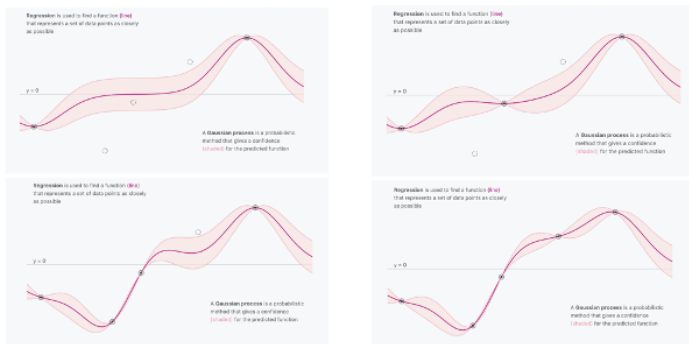


# I – Data: Active learning

## Gaussian processes

$$y_A | y_B = y_B \sim \mathcal{N}(\mu_A + \Sigma_{AB} \Sigma_{BB}^{-1} (y_B - \mu_B), \Sigma_{AA} - \Sigma_{AB} \Sigma_{BB}^{-1} \Sigma_{BA})$$

$$f_* | (Y_1 = y_1, \dots, Y_n = y_n, \mathbf{x}_1, \dots, \mathbf{x}_n, \mathbf{x}_t) \sim \mathcal{N}(K_*^\top K^{-1} y, K_{**} - K_*^\top K^{-1} K_*)$$



MD  
&  
QoI

## Physics informed

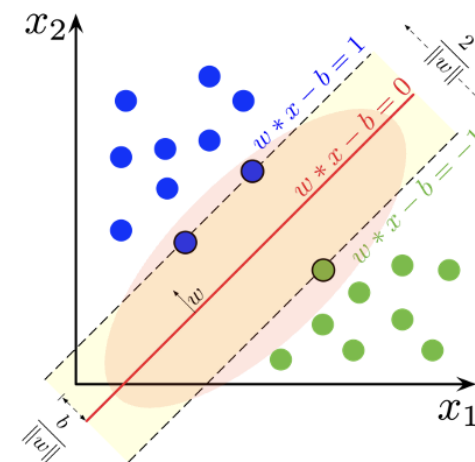
$$u(x, \mu_1, \dots, \mu_P) \approx \sum_{i=1}^N X_i(x) M_i^1(\mu_1) \cdots M_i^P(\mu_P)$$

$$X_i(x), i = 1, \dots, N$$

DEIM

## Classification

SVM



Entropy-based  
sampling

## Physics augmented

$$(\mathbf{K} + \Delta \mathbf{K})(\mathbf{U} + \Delta \mathbf{U}) = \mathbf{F}$$

$$\Delta \mathbf{U} = (\mathbf{K} + \Delta \mathbf{K})^{-1} \mathbf{F} - \mathbf{U}$$

$$\hat{x}_{\Delta \mathbf{K}} = \mathbf{X}_j \rightarrow \max_j |\Delta U_j|$$

# II – Modelling from data

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## □ Fundamental laws:

- Mass, Momentum, Energy, ... conservation
- Entropy production
- Objectivity

## □ Conservation vs symmetries:

- Noether theorem

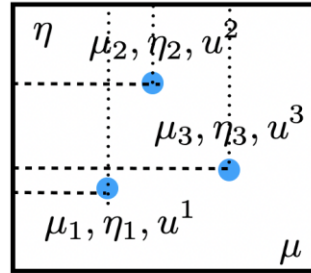
## □ Other properties:

- Stability, Convexity, ...

## □ Phenomenological constitutive equations

# II – Modelling from data

LINEAR



Linear approximation

$$u(\mu, \eta) = a + b\mu + c\eta$$

Beyond linear approximations

$$[1, \mu, \eta, \mu^2, \mu\eta, \eta^2, \mu^2\eta, \mu^2\eta^2, \mu\eta^2]$$

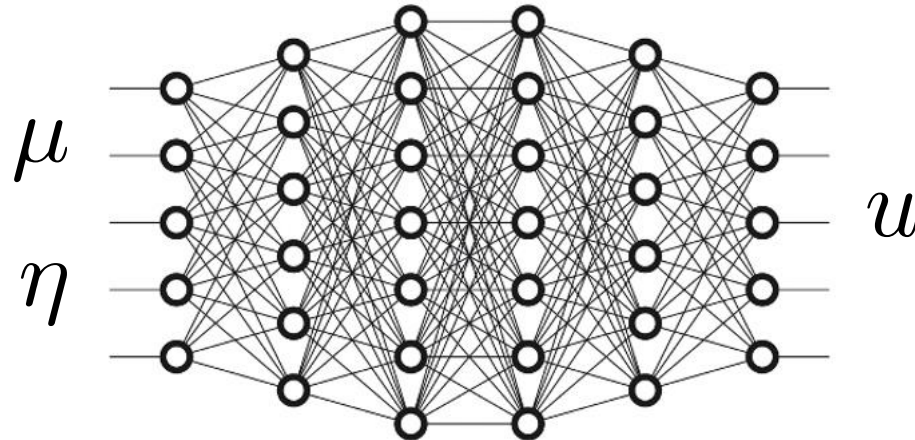
$$u(\mu, \eta) = \sum_{i=1}^N c_i \mathcal{F}_i(\mu, \eta)$$

Regularization by sparsity

$$\sum_{j=1}^3 \|u(\mu_j, \eta_j) - u^j\|_2^2 + \lambda \sum_{i=1}^N |c_i|$$

Regularized  
Polynomial  
Regression  
**-RPR-**

NONLINEAR



Nonlinear  
Regression  
**-NLR-**

## II – Remark: Regression/Approximation

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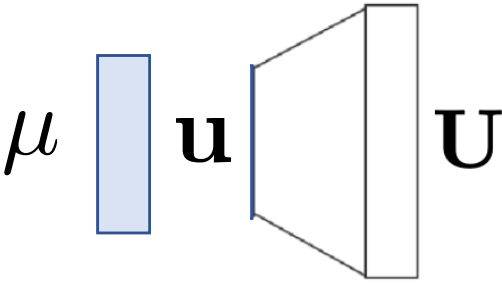
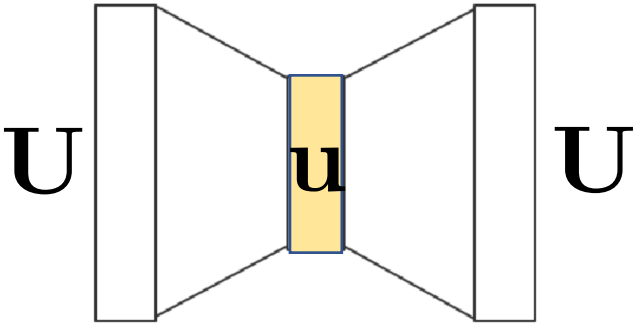
Linear regression-based linear approximation:

$$u = a + b\mu + c\eta$$

Linear regression-based nonlinear approximation:

$$u = a + b\mu + c\eta + d\mu\eta + e\mu^2 + \dots$$

# II – Modelling at the latent space

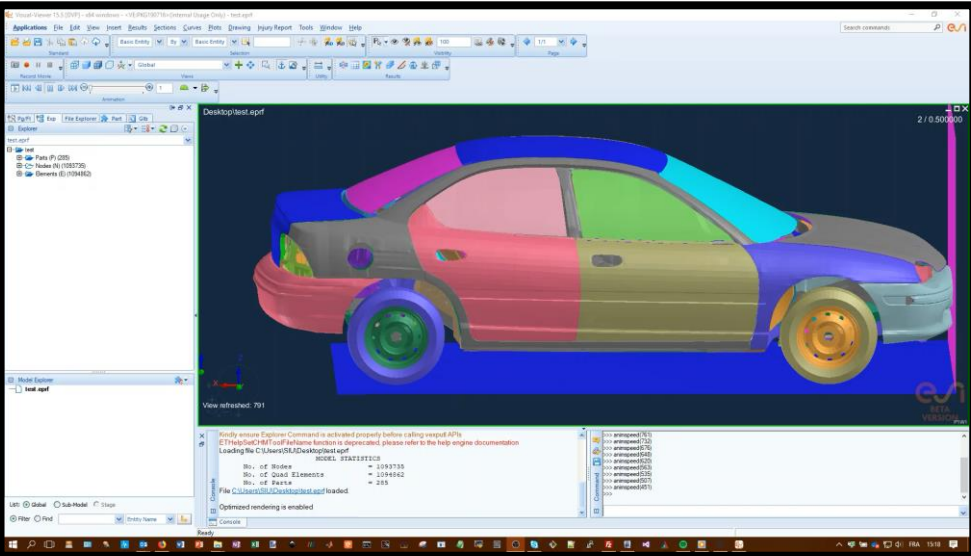
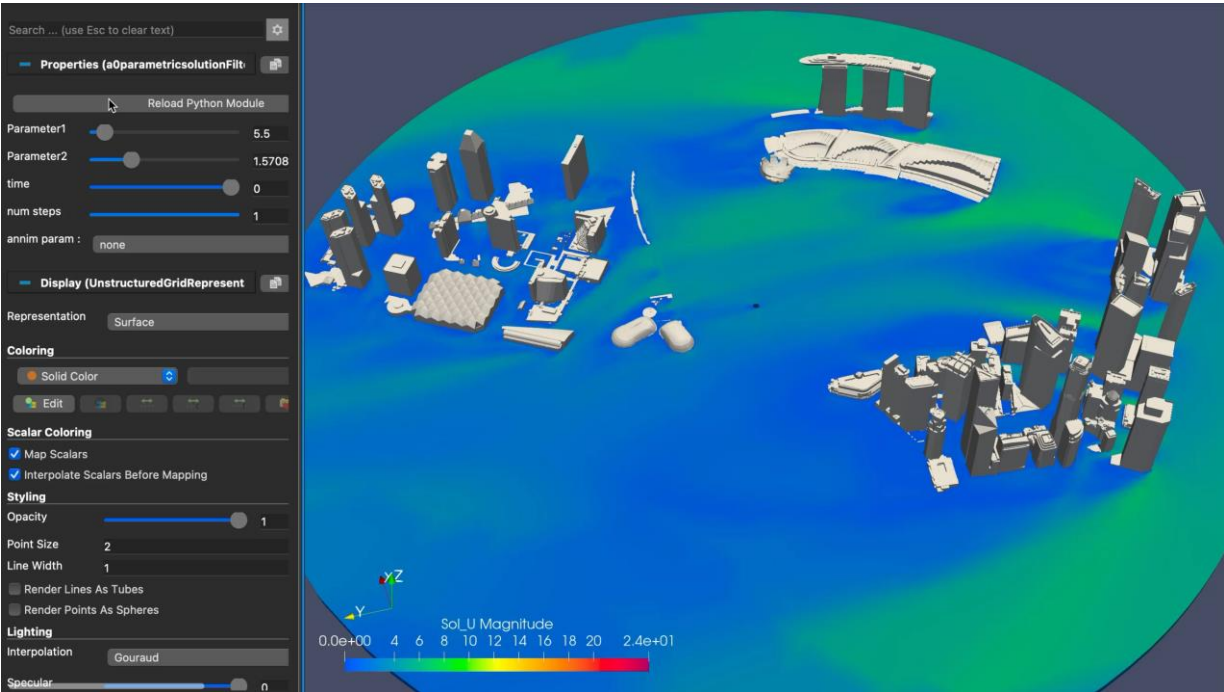
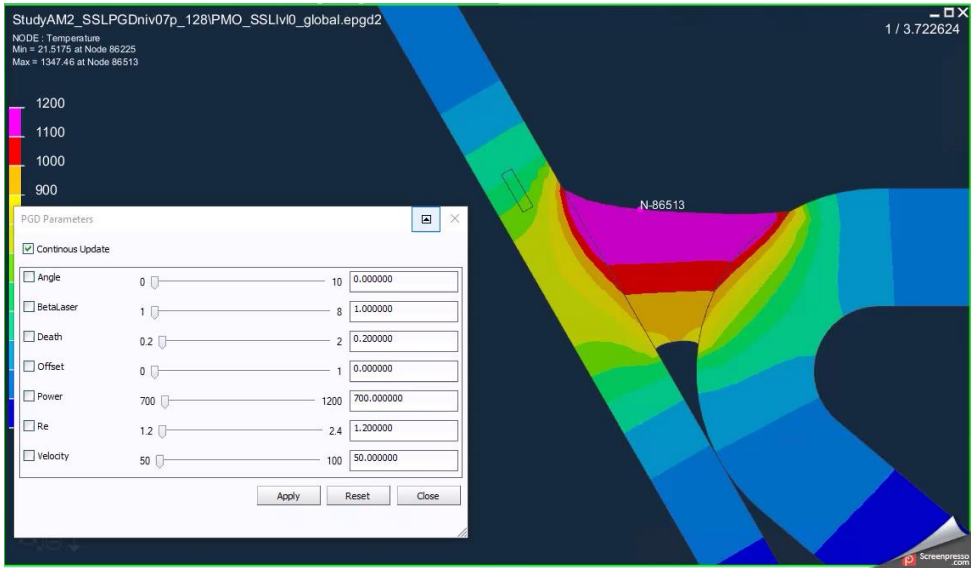


|                                                                                       |     |                                                                                       |     |
|---------------------------------------------------------------------------------------|-----|---------------------------------------------------------------------------------------|-----|
|    | PCA |    | RPR |
|    | PCA |    | NLR |
|   | AE  |   | RPR |
|  | AE  |  | NLR |

# II – Linear-Linear

PCA

RPR



# II – Linear-Linear



## Physics-aware interaction between virtual and physical objects in Mixed Reality

A. Badías, D. González, I. Alfaro, F. Chinesta, E. Cueto



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## Reduced-order modelling for augmented reality

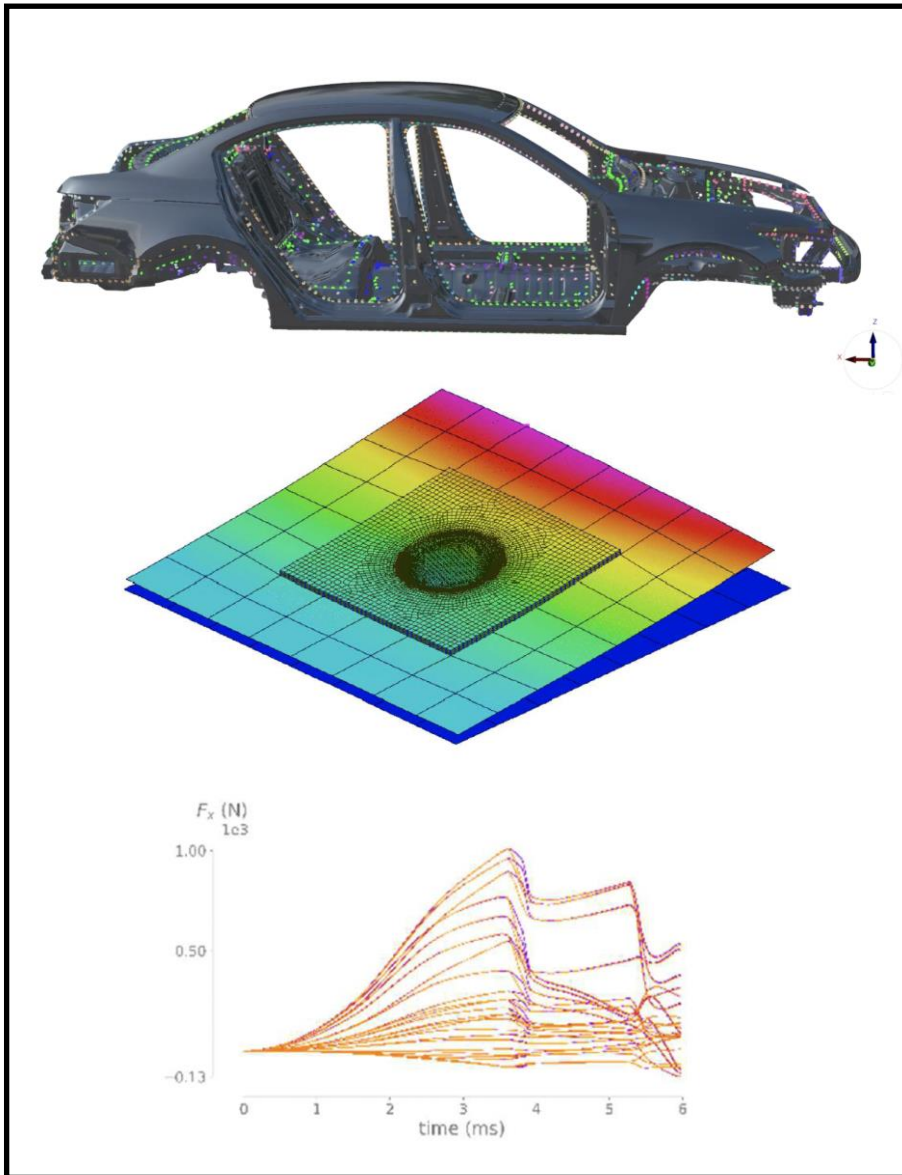
Rubber Piece

A. Badías, I. Alfaro, D. González, F. Chinesta and E. Cueto

✉ abadias@unizar.es



# II – Nonlinear-Nonlinear



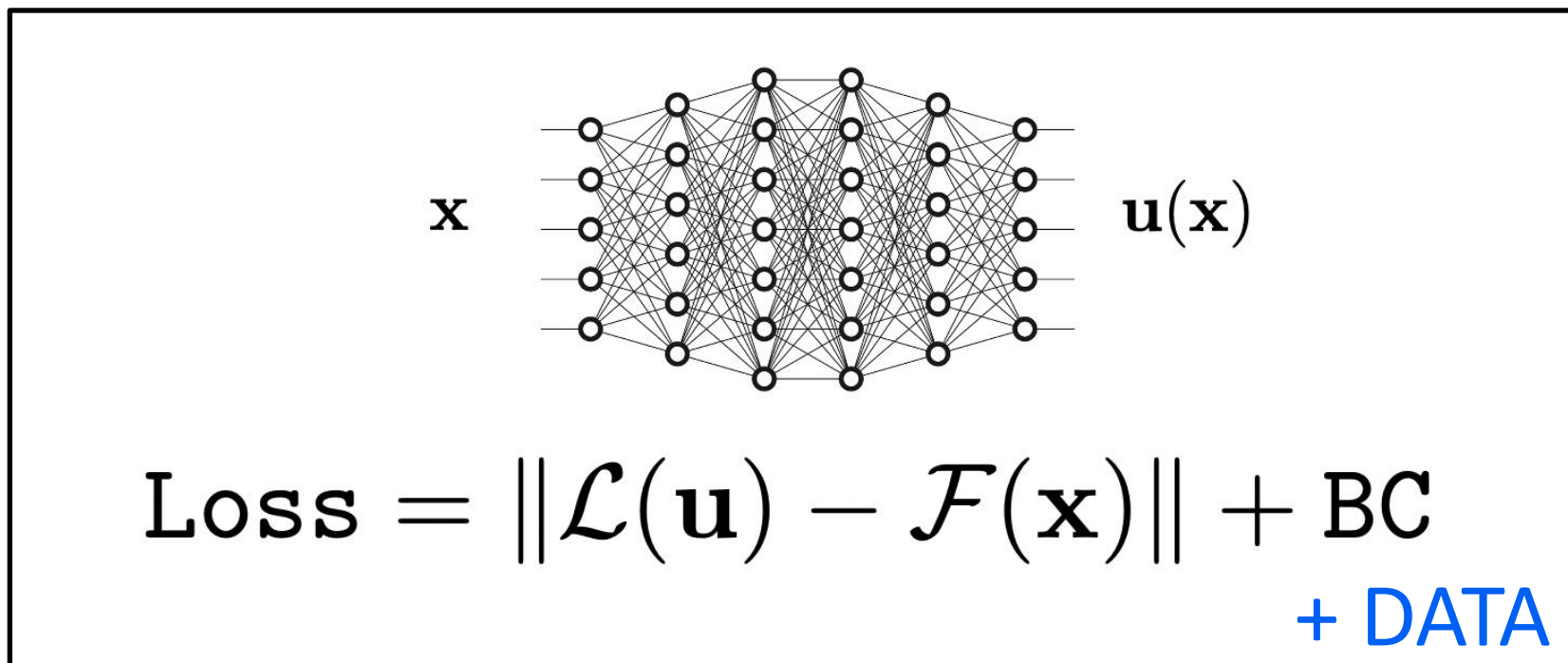
## Physically sound, self-learning digital twins for sloshing fluids

B. Moya, I. Alfaro, D. González, F. Chinesta, E. Cueto

# III – Hybridation

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## Physics-informed



Physics-Augmented: Physics-based + Data-driven

# IV - Numerical technologies

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Regularized polynomial regressions, GP, DT, RF, SVR, ...

CNN

GNN

rNN, LSTM, ResNET, NeuralODE, DeepONet, Reservoir computing, ...

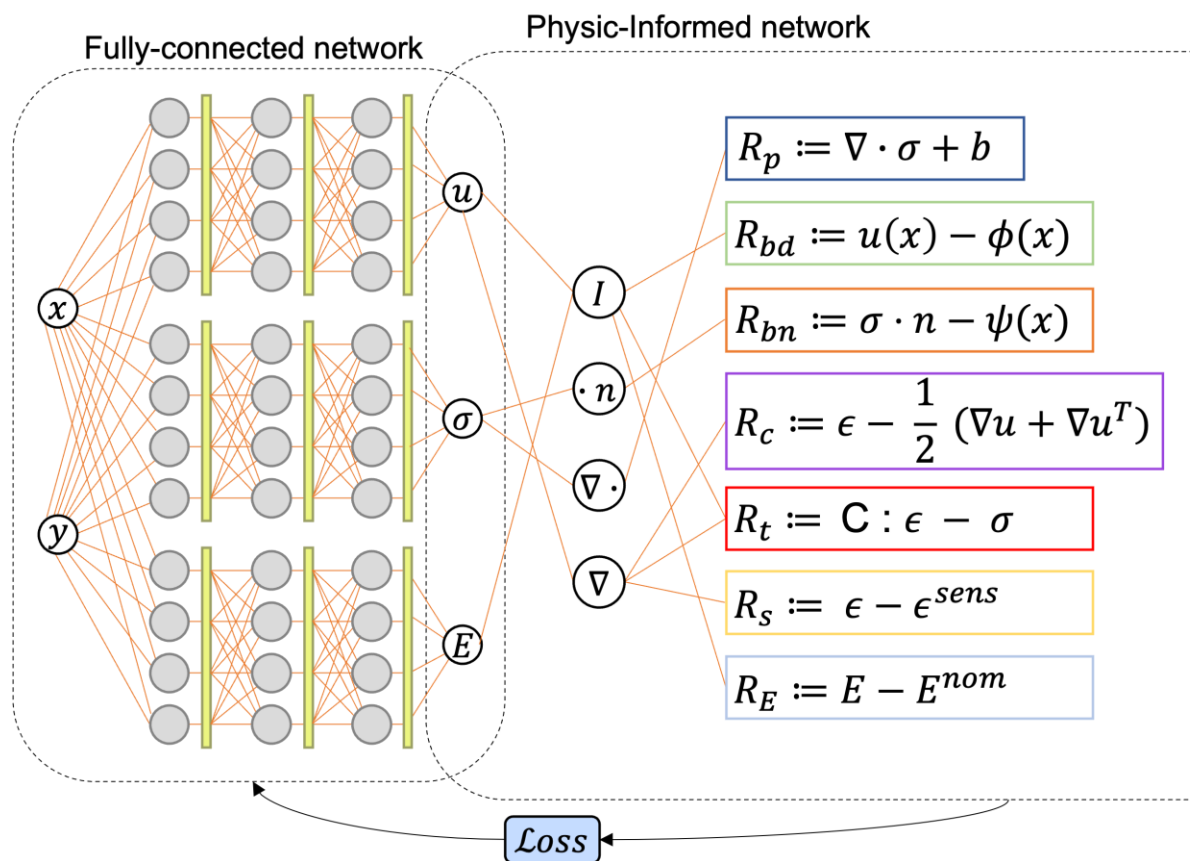
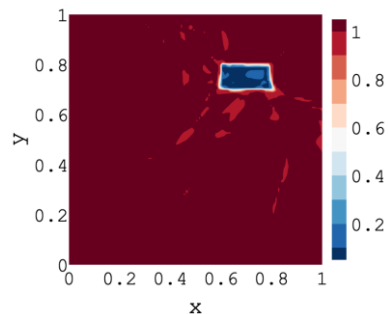
GAN

Transformers

Autoencoders

PINN, SPNN, PANN, ...

# PINN



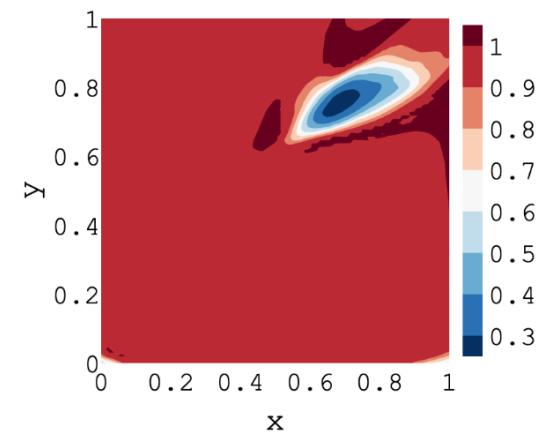
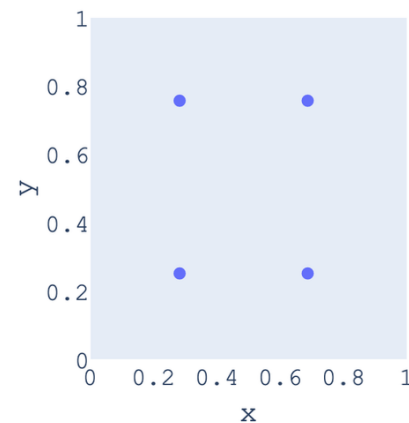
$$\mathcal{L}_d = \frac{1}{N_d} \sum_i^{N_d} |\nabla \cdot \hat{\sigma}(\mathbf{x}^i) - \mathbf{b}(\mathbf{x}^i)|^2$$

$$\mathcal{L}_b = \frac{1}{N_{dc}} \sum_j^{N_{dc}} |\hat{u}(\mathbf{x}^j) - \phi(\mathbf{x}^j)|^2 + \frac{1}{N_{nn}} \sum_l^{N_{nn}} |\hat{\sigma}(\mathbf{x}^l) \cdot \mathbf{n} - \psi(\mathbf{x}^l)|^2$$

$$\mathcal{L}_f = \frac{1}{N_d} \sum_i^{N_d} |\hat{\sigma}(\mathbf{x}^i) - C : \epsilon(\mathbf{x}^i)|$$

$$\mathcal{L}_{dat} = \frac{1}{N_{sens}} \sum_j^{N_{sens}} |\epsilon(\mathbf{x}^j) - \epsilon^{sens}(\mathbf{x}^j)|^2$$

$$\mathcal{L}_c = \frac{1}{N_d} \sum_i^{N_d} |\hat{E}(\mathbf{x}^i) - E^{nom}(\mathbf{x}^i)|$$



# TINN

$$\dot{\mathbf{z}}_t = \mathbf{L}(\mathbf{z}_t) \nabla E(\mathbf{z}_t) + \mathbf{M} \nabla S(\mathbf{z}_t), \quad \mathbf{z}(0) = \mathbf{z}_0$$



Poisson matrix:  
reversibility

Friction matrix:  
irreversibility

with

$$\begin{aligned} \mathbf{L}(\mathbf{z}) \cdot \nabla S(\mathbf{z}) &= \mathbf{0}, \\ \mathbf{M}(\mathbf{z}) \cdot \nabla E(\mathbf{z}) &= \mathbf{0}. \end{aligned}$$

$$\frac{\mathbf{z}_{n+1} - \mathbf{z}_n}{\Delta t} = \mathbf{L}(\mathbf{z}_{n+1}, \mathbf{z}_n) \text{DE}(\mathbf{z}_{n+1}, \mathbf{z}_n) + \mathbf{M}(\mathbf{z}_{n+1}, \mathbf{z}_n) \text{DS}(\mathbf{z}_{n+1}, \mathbf{z}_n)$$

$$\boldsymbol{\mu} = \{\mathbf{L}, \mathbf{M}, \text{DE}, \text{DS}\} = \arg \min_{\boldsymbol{\mu}^*} \|\mathbf{z}(\boldsymbol{\mu}) - \mathbf{z}^{\text{meas}}\|$$

with

$$\begin{aligned} \text{DE} &= \mathbf{A}\mathbf{z} \\ \text{DS} &= \mathbf{B}\mathbf{z} \end{aligned}$$

## Physically sound, self-learning digital twins for sloshing fluids

B. Moya, I. Alfaro, D. González, F. Chinesta, E. Cueto

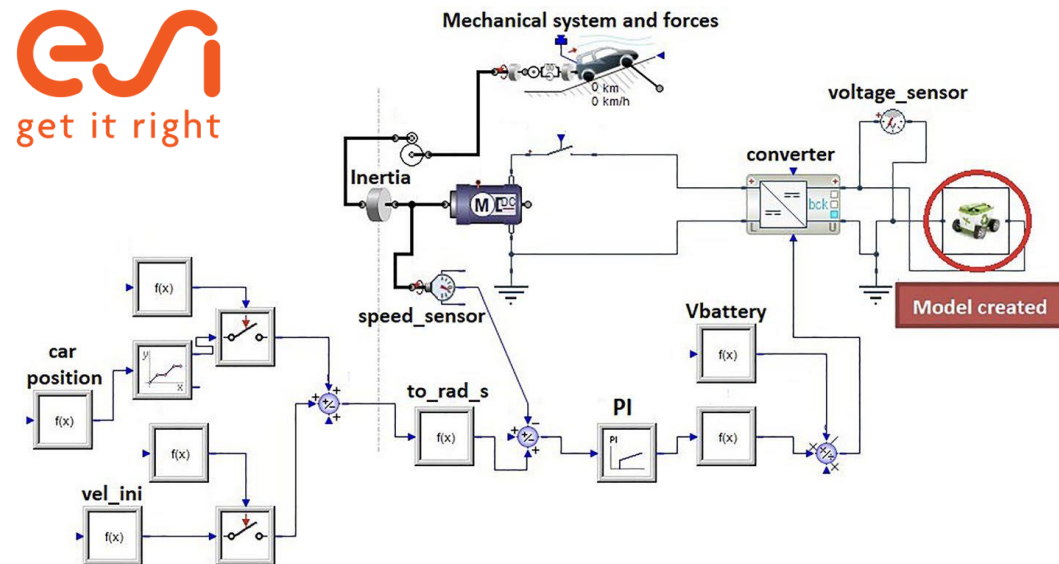
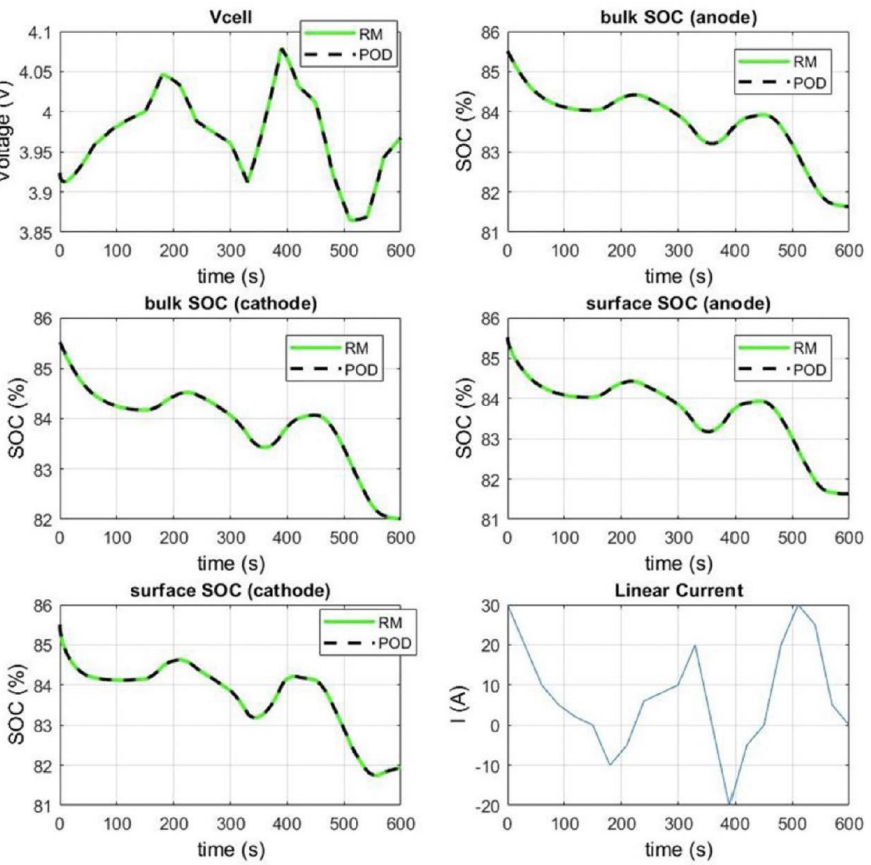


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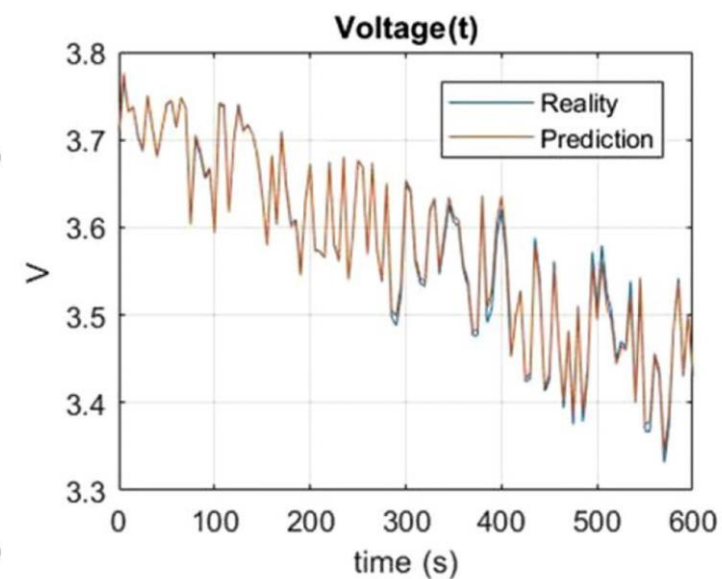
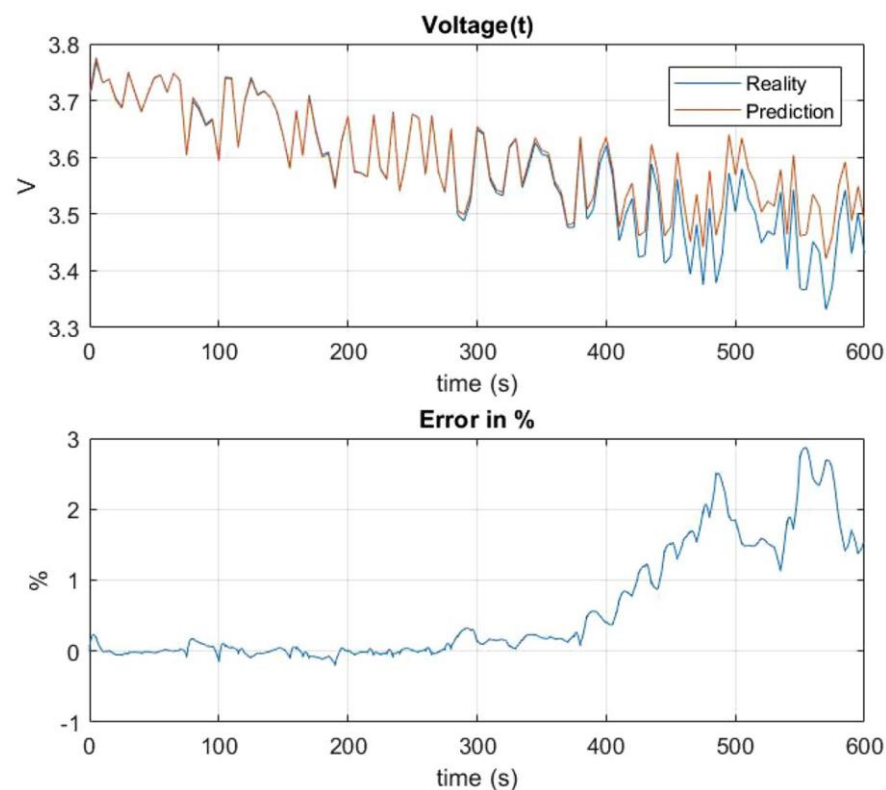
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# Agenda

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3. Applications

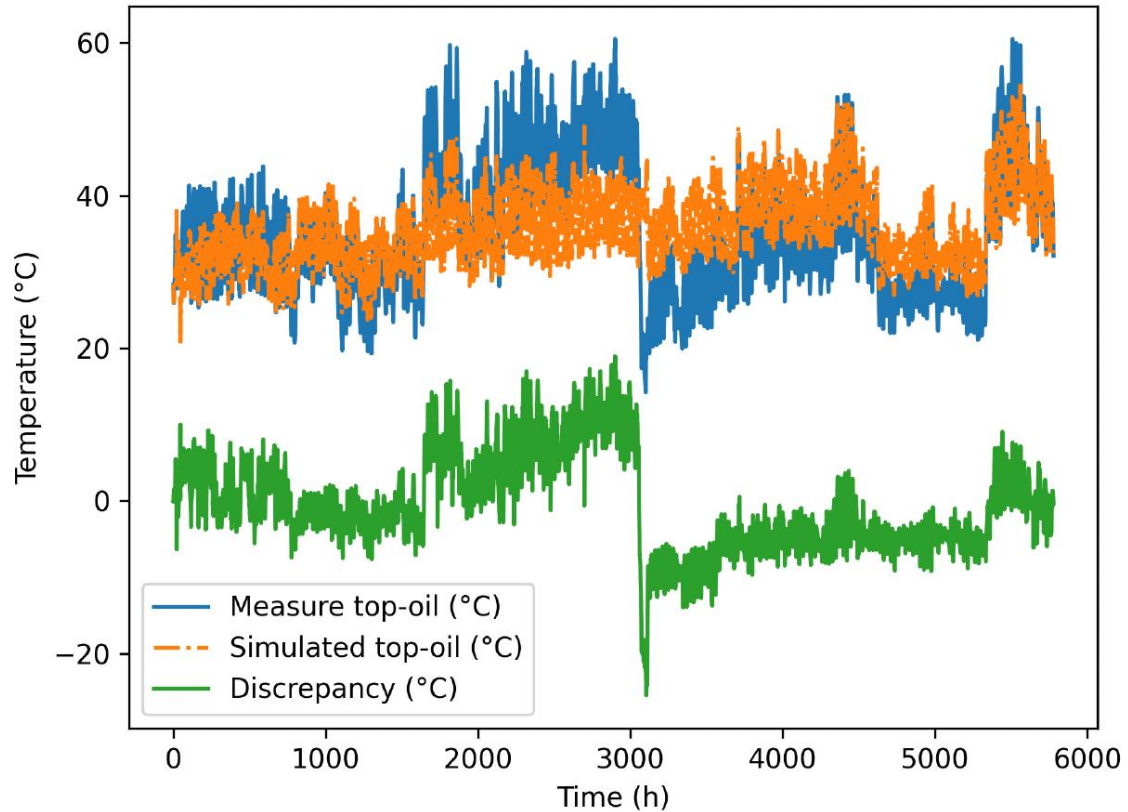


# HYBRID

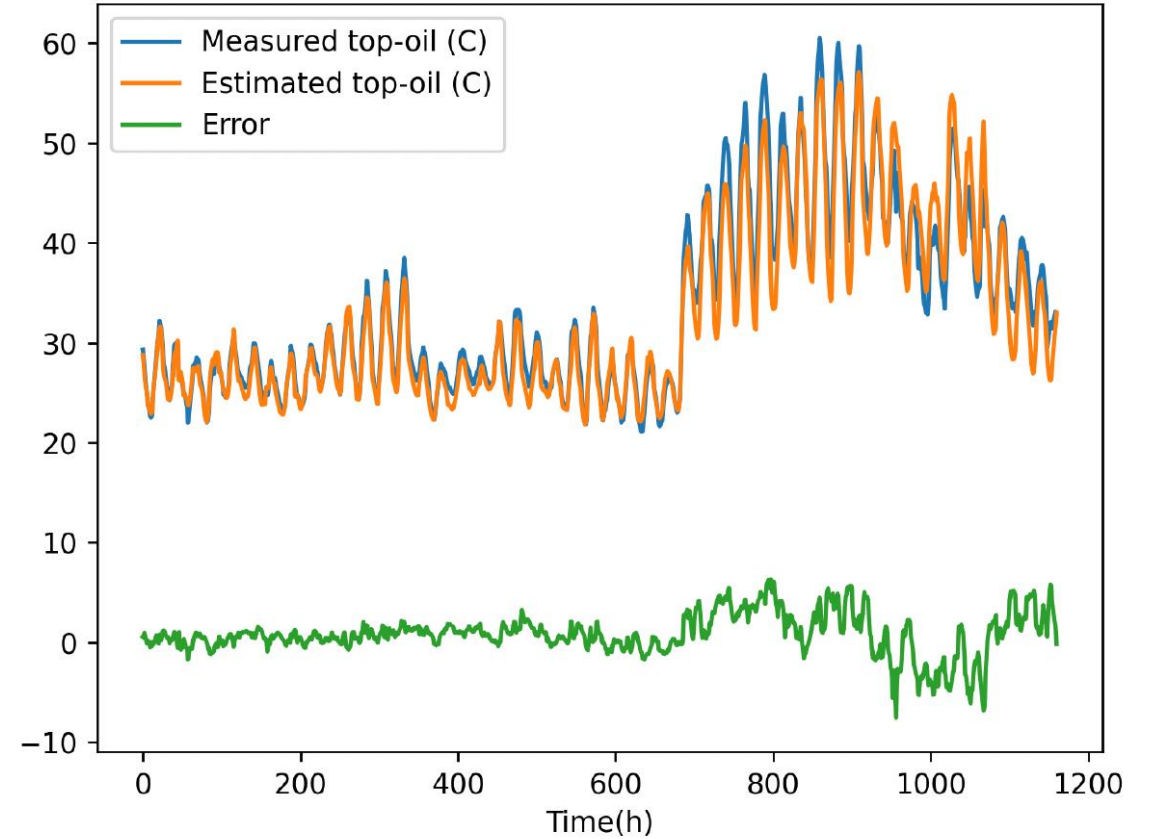


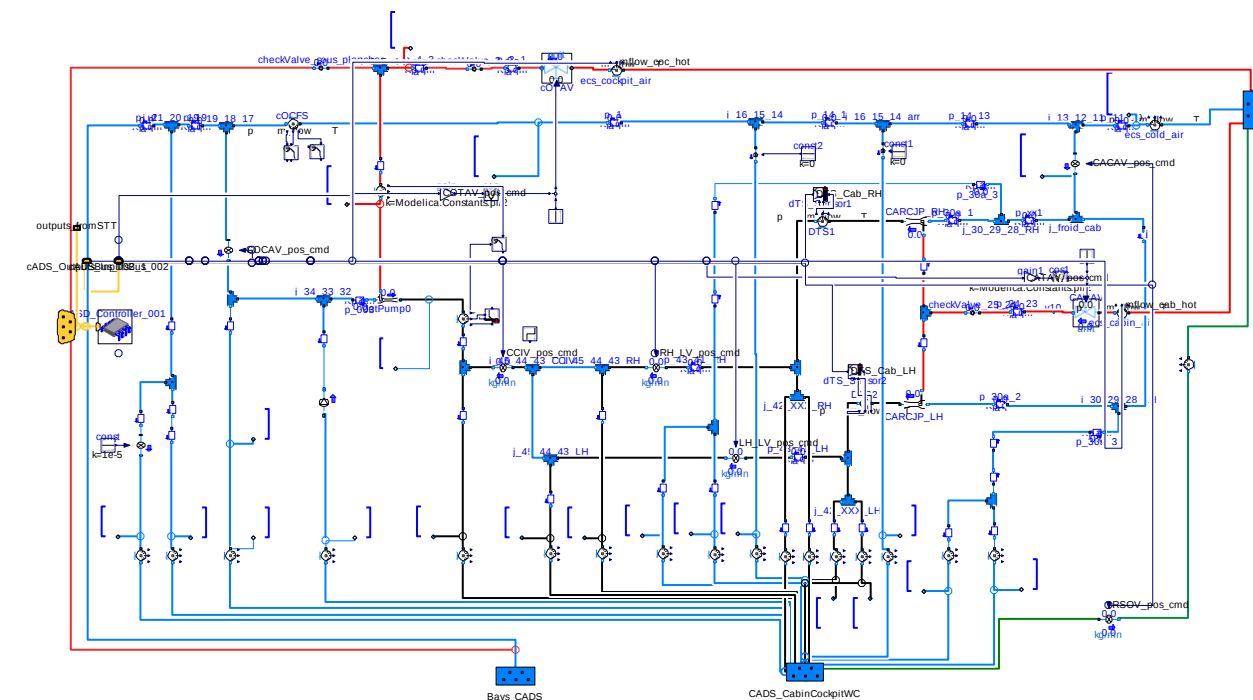
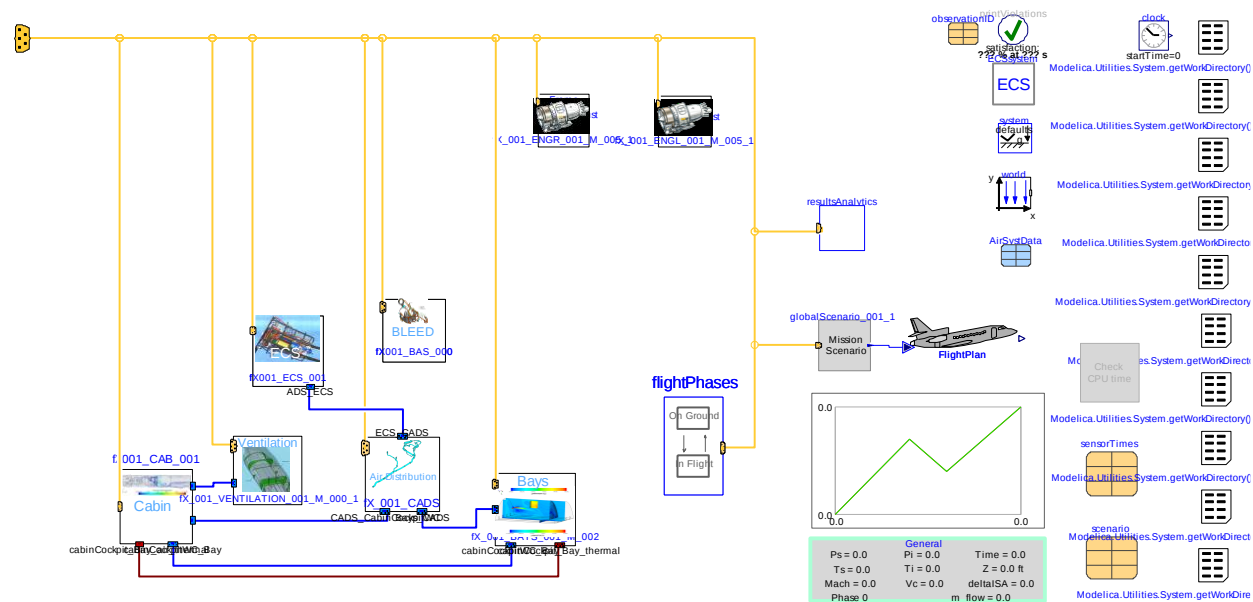
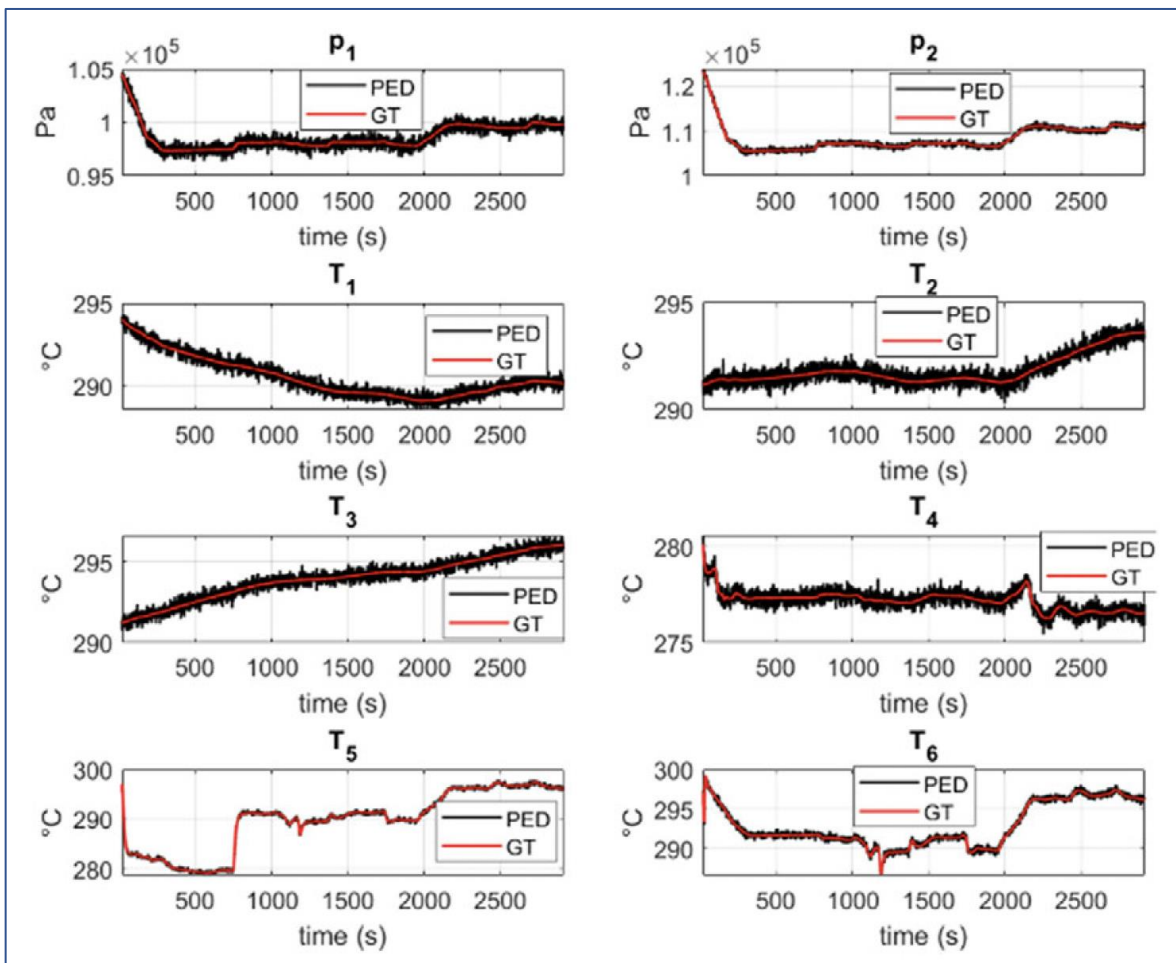
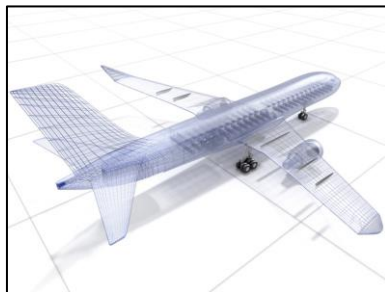


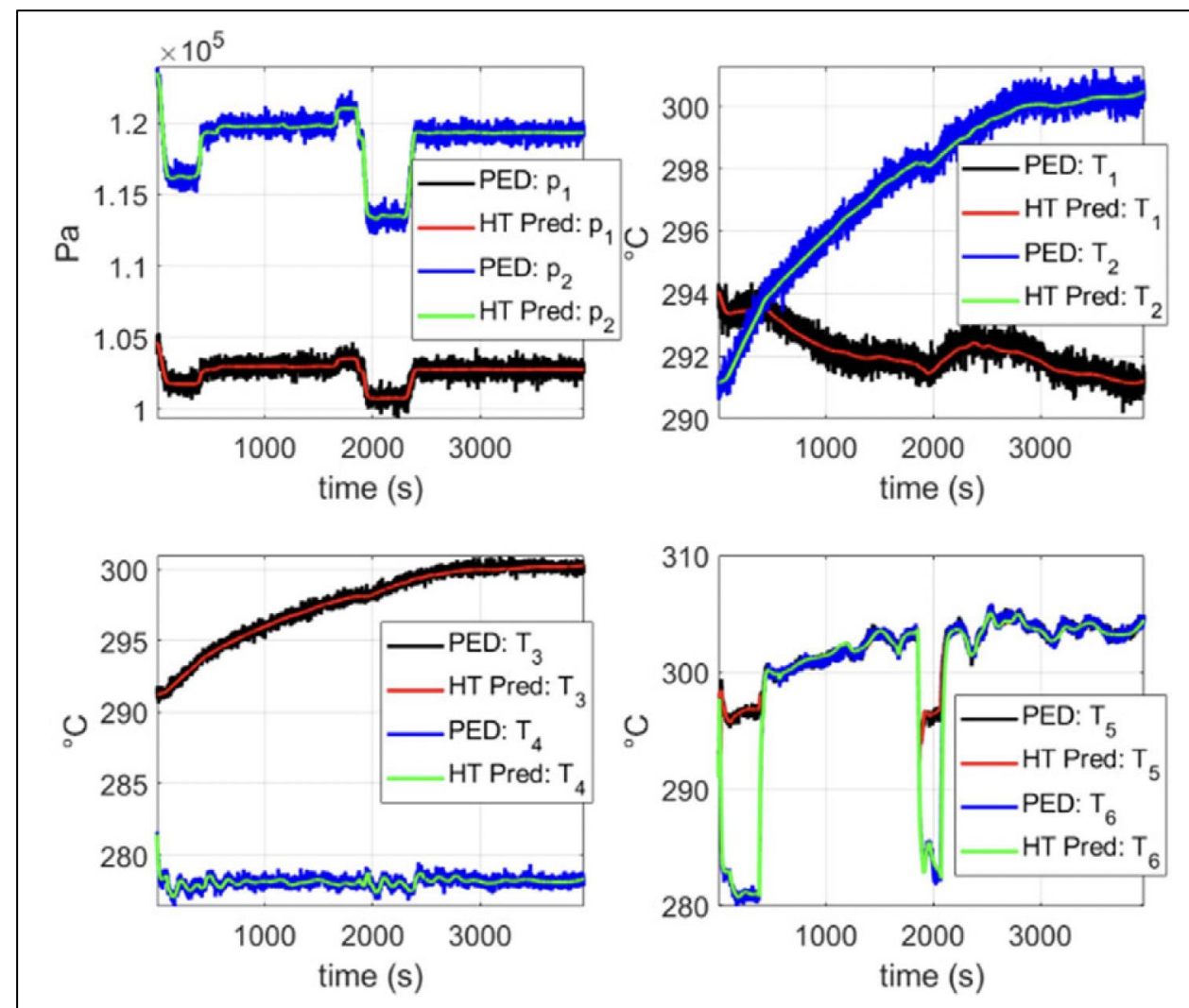
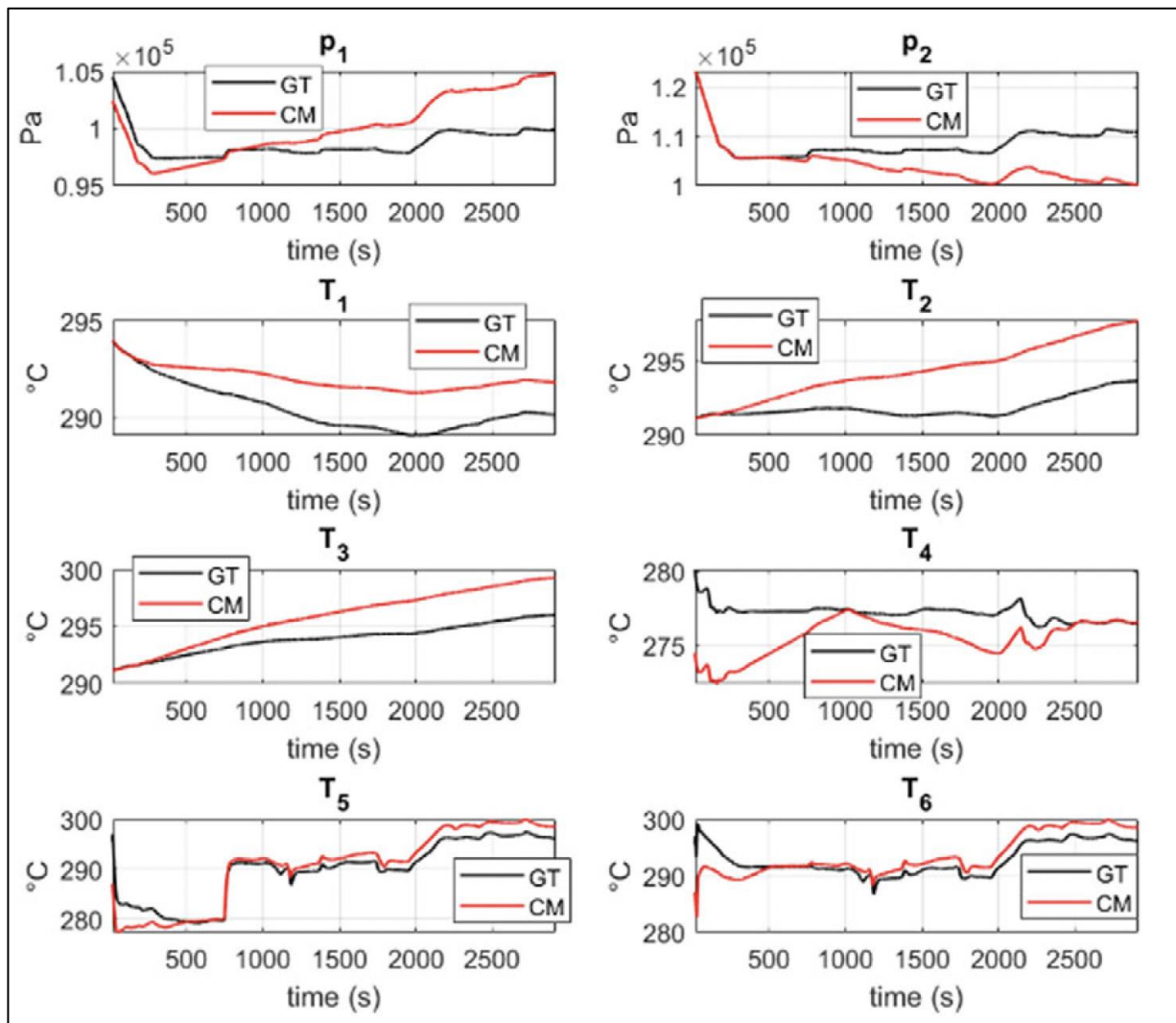
Le réseau  
de transport  
d'électricité

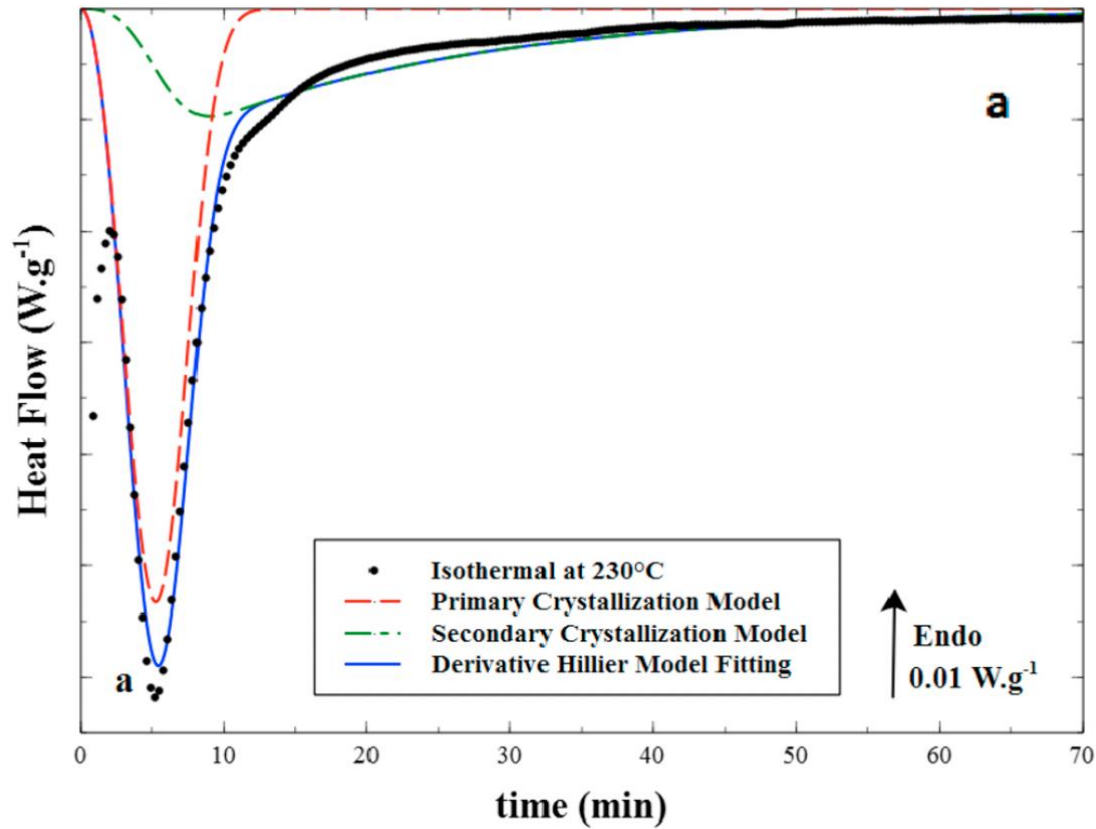
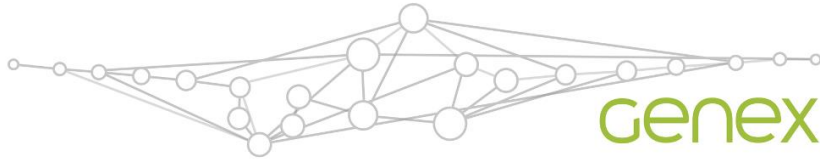


Testing+integration: HT Oil temperature estimation for a RTE transformer





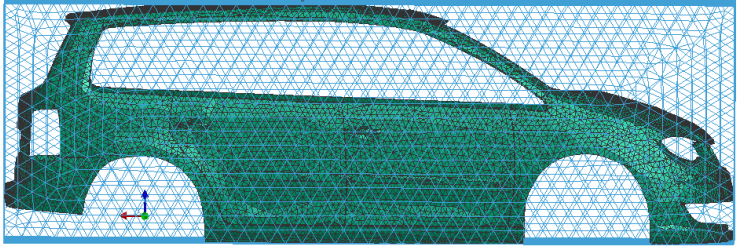




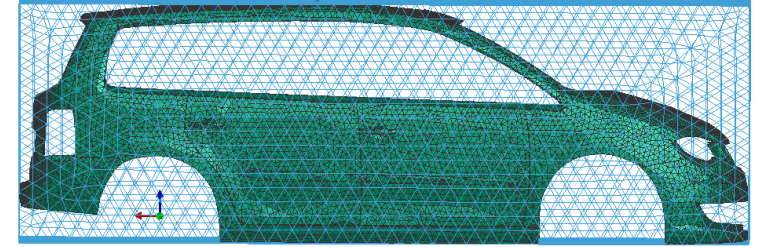
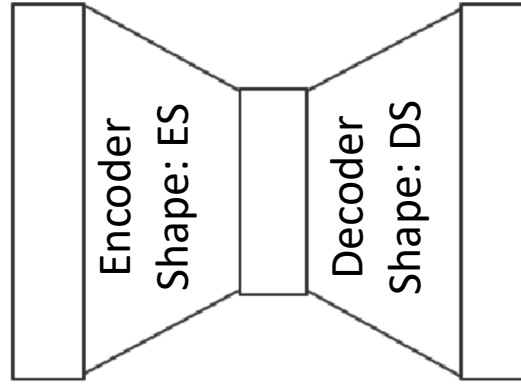
Hybrid modelling of  
local kinetics

$$\frac{d\alpha}{dt} = f(t; \mathbf{p}) + \mathbf{g}(\mathbf{t}; \mathbf{p})$$

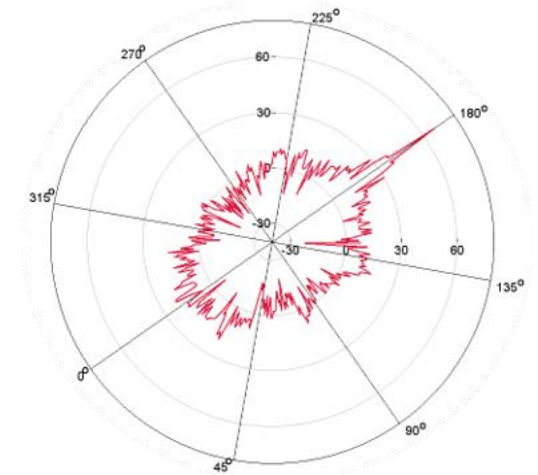
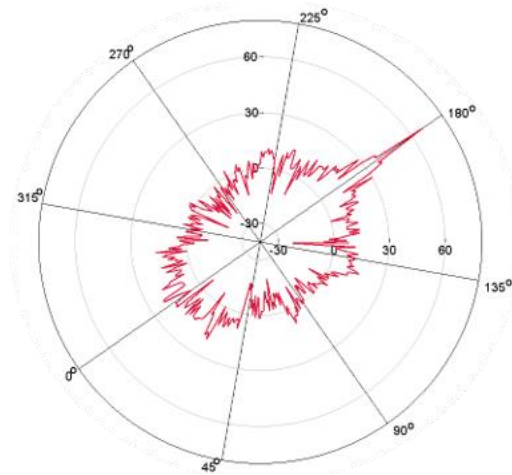
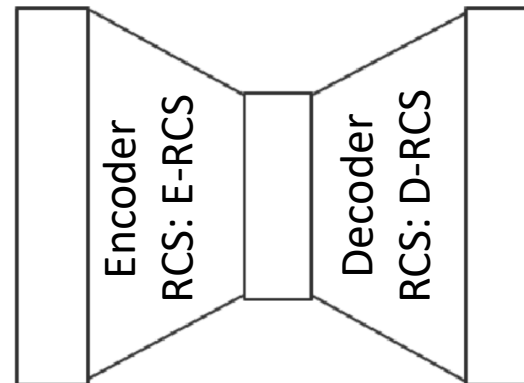
Model      Data-Driven  
correction

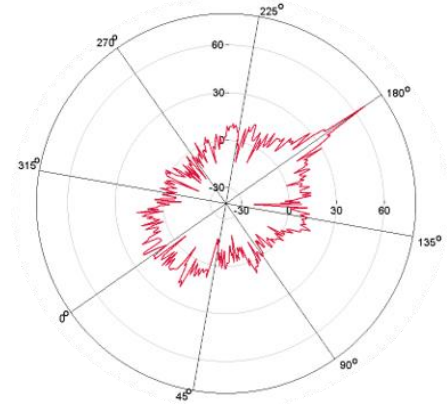
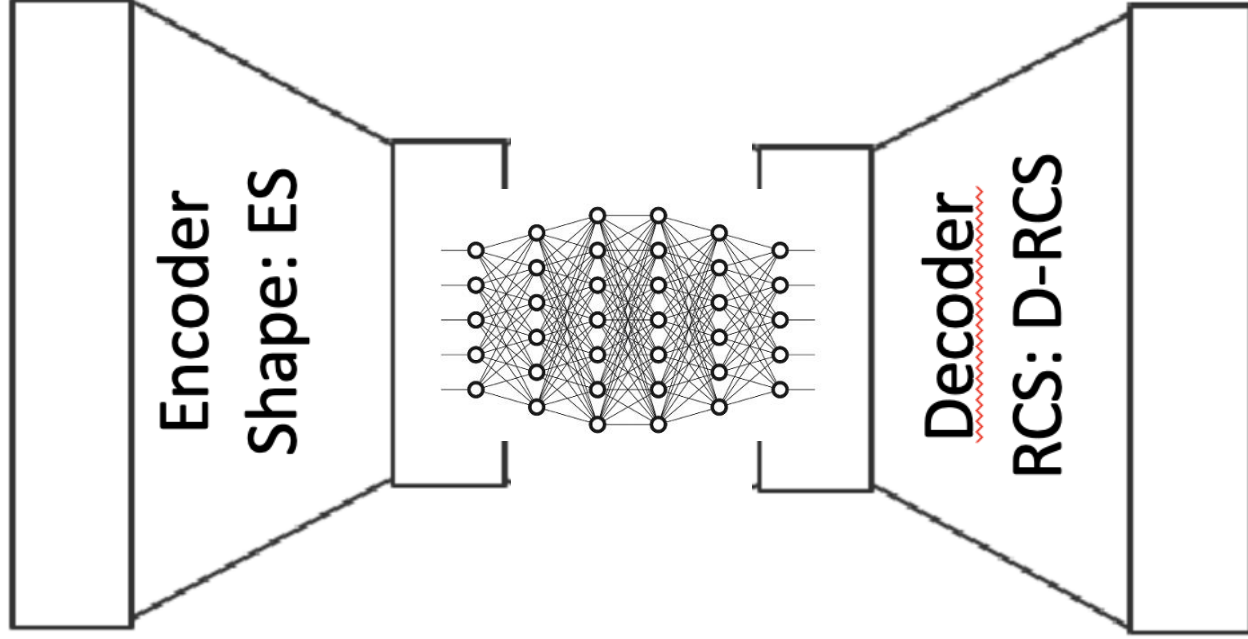
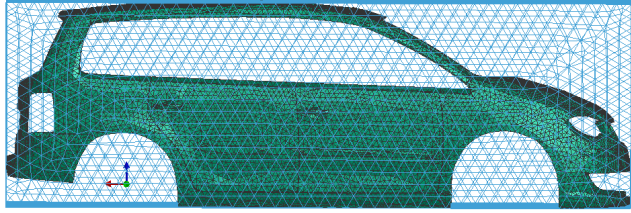


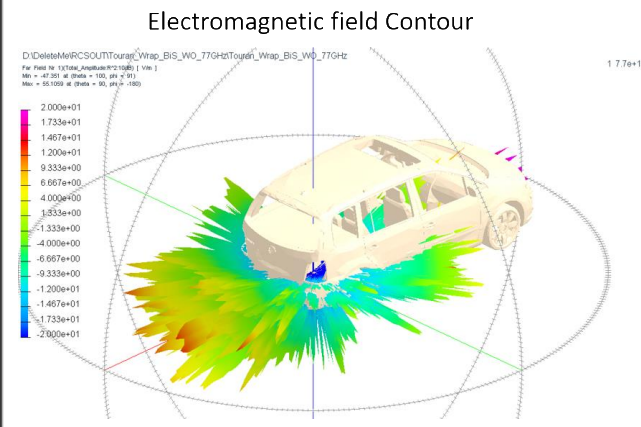
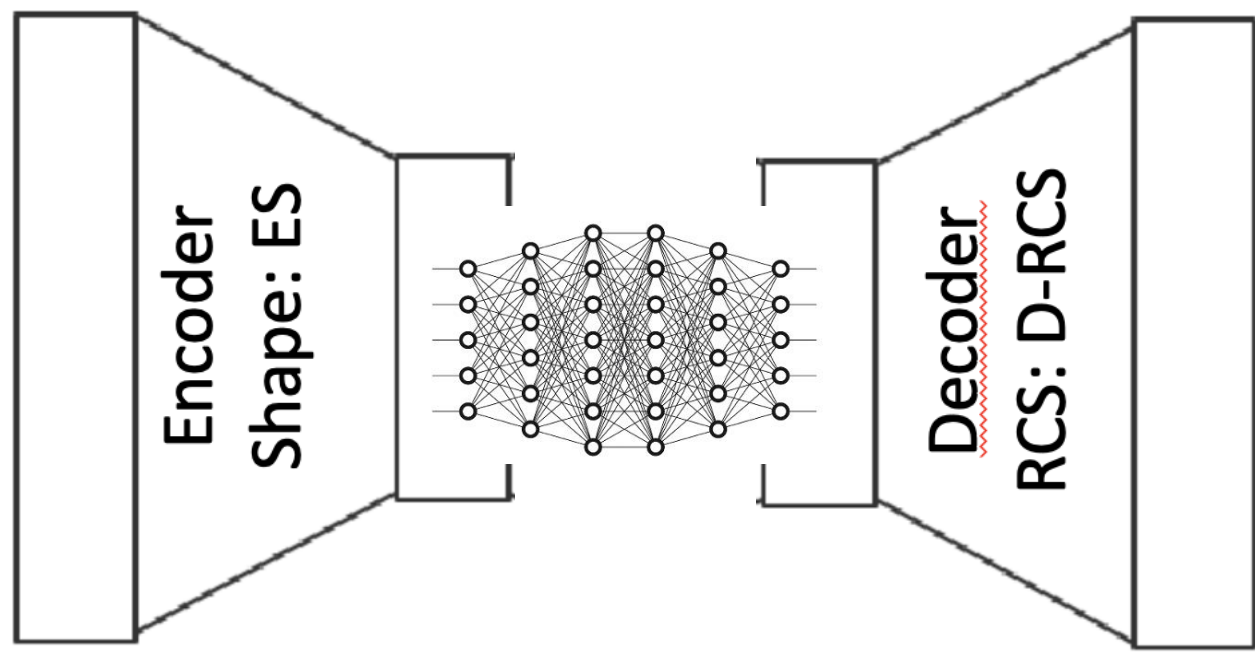
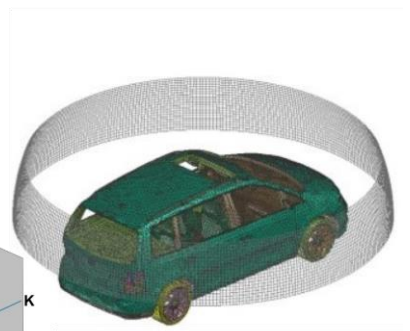
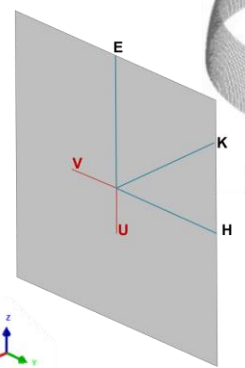
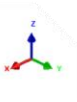
## Autoencoder of Shapes: AE-S



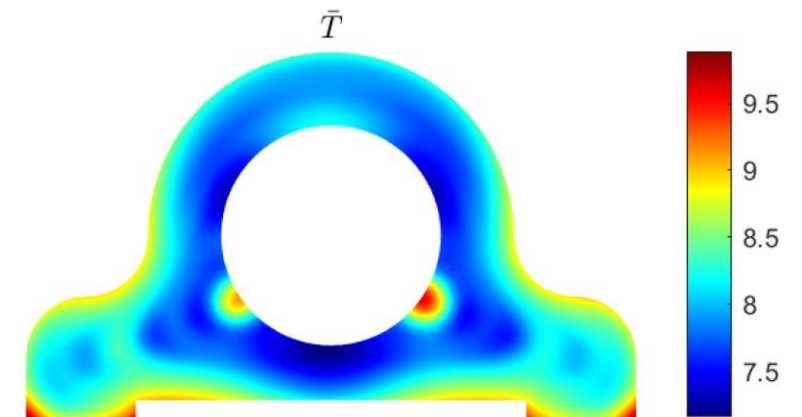
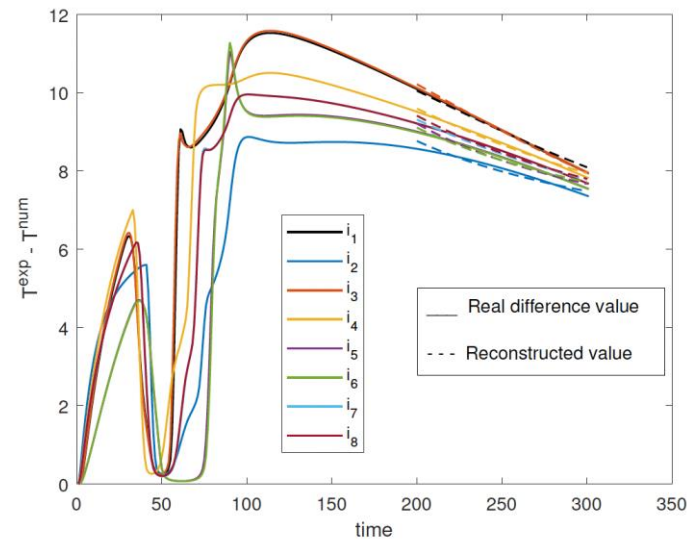
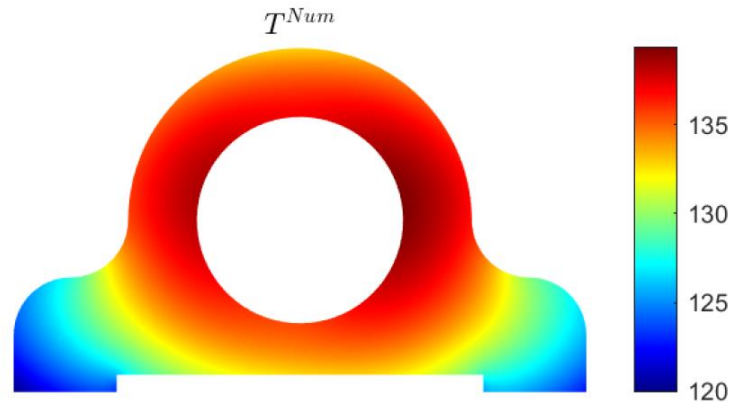
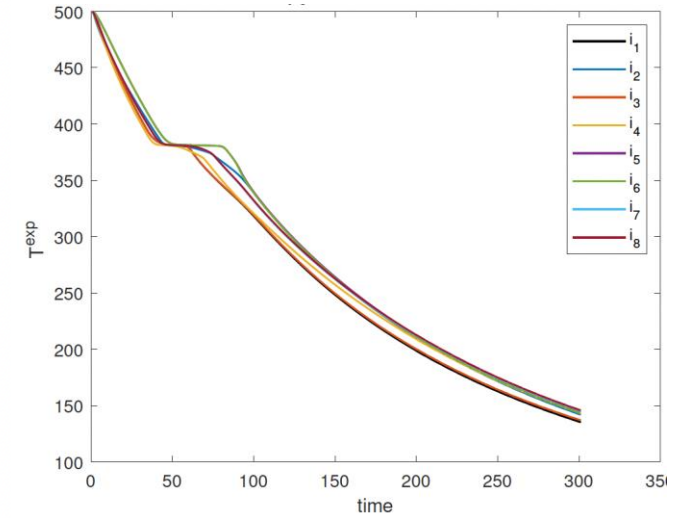
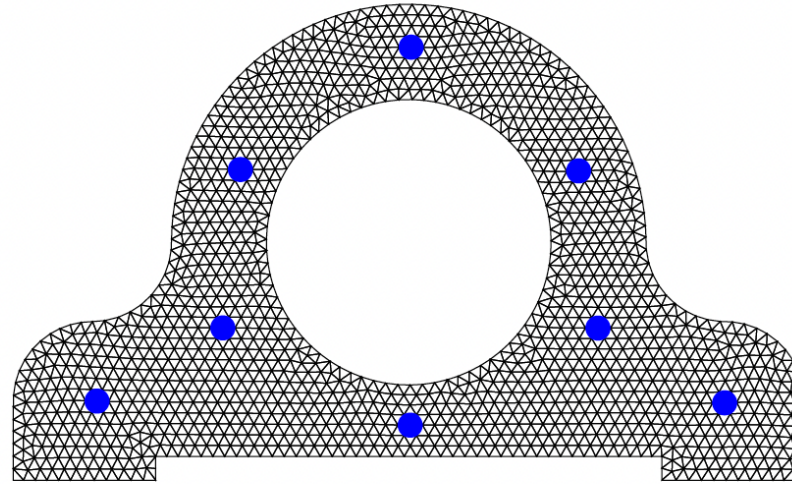
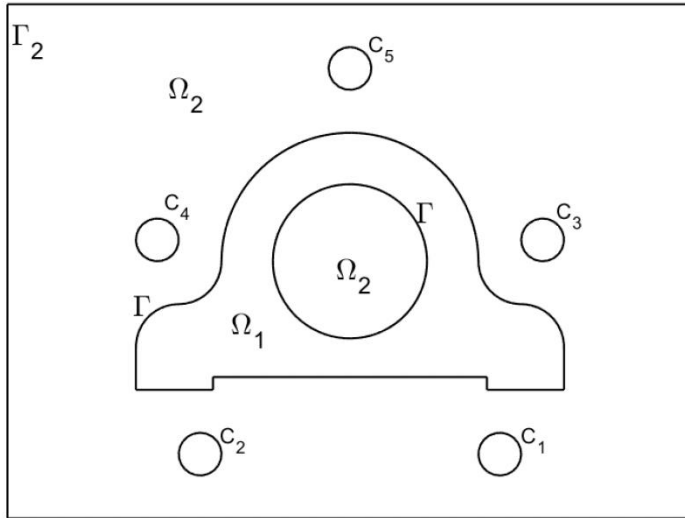
## Autoencoder of RCS: AE-RCS

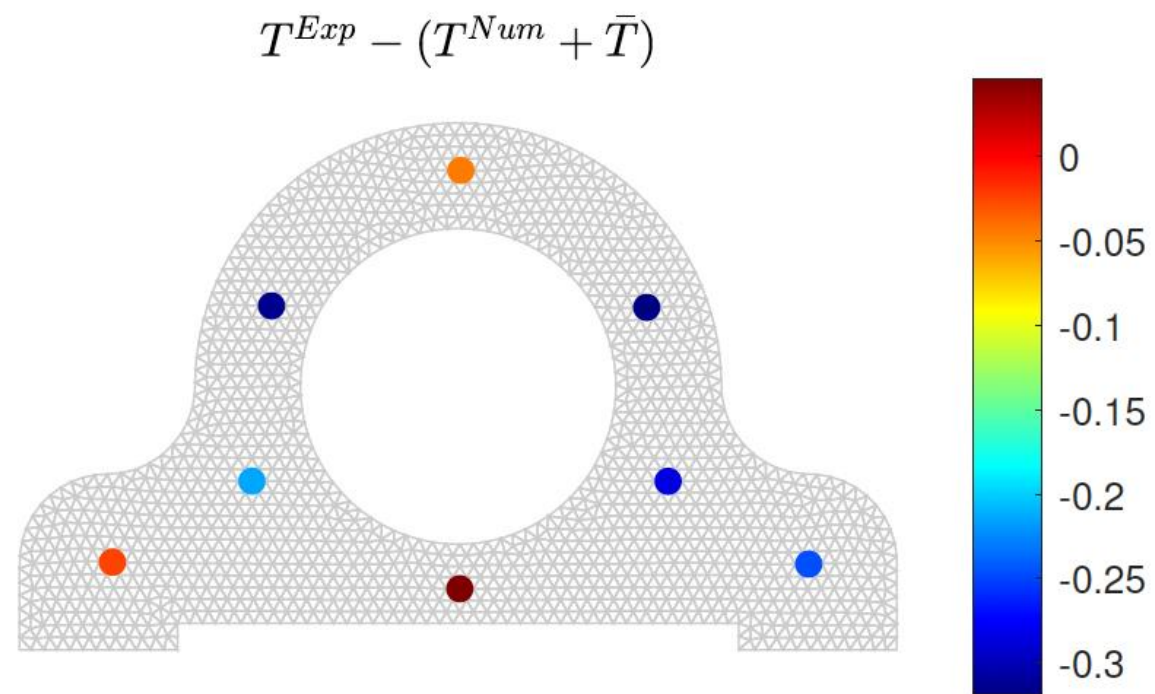
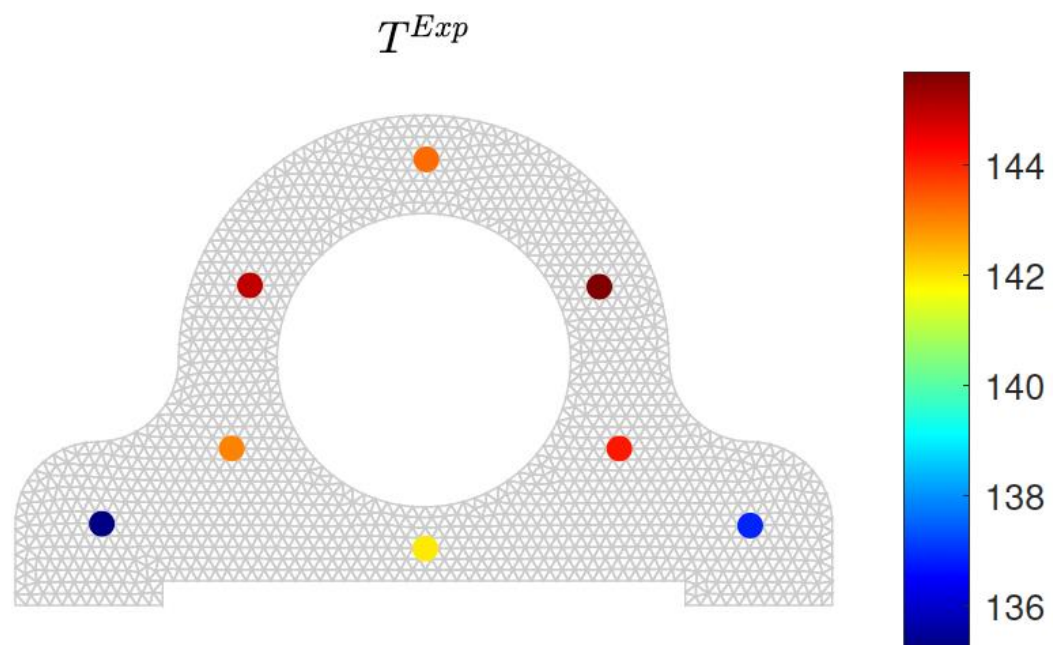
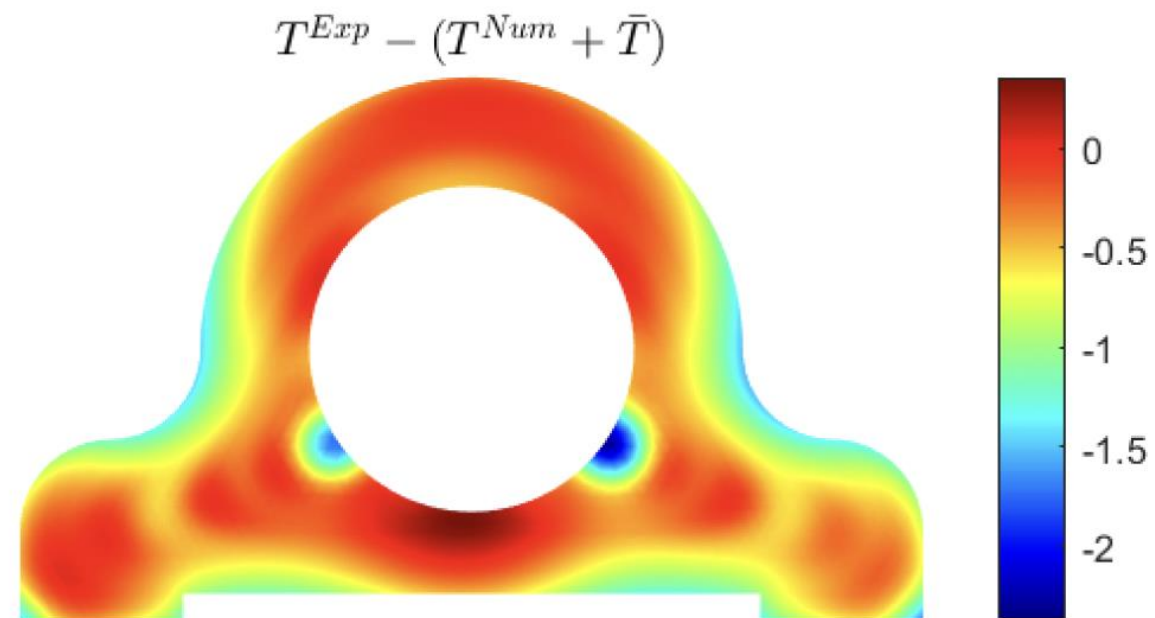
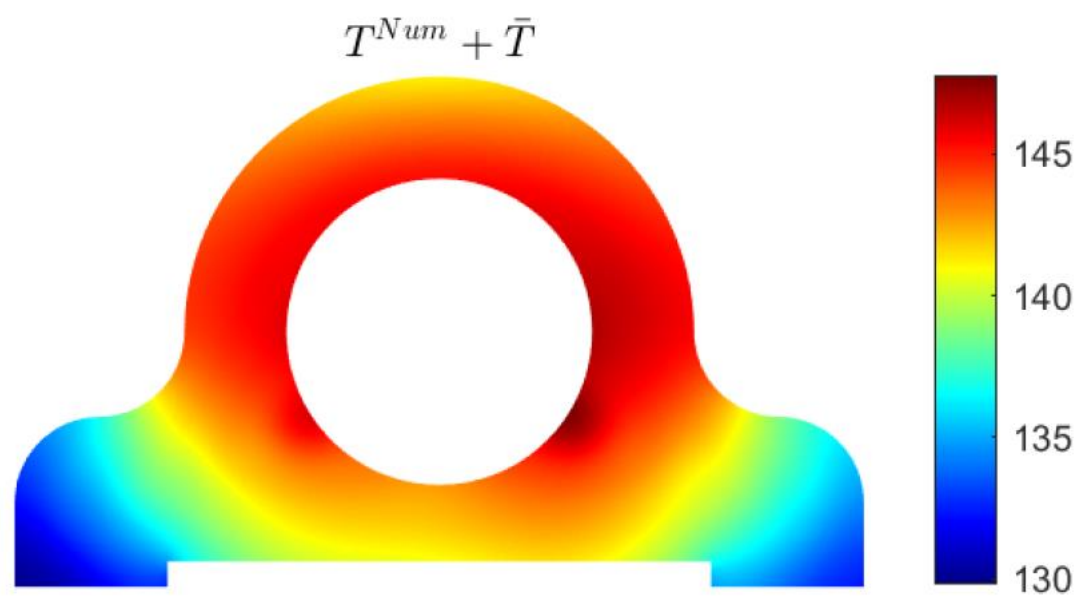






# Correcting beyond the observation





# Cheap simulation + learnt correction

## Predicting high-fidelity data from coarse-mesh Computational Fluid Dynamics corrected using Hybrid Twins based on Optimal Transport

Sergio Torregrosa<sup>a,b,\*</sup>, Victor Champaney<sup>c</sup>, Amine Ammar<sup>d</sup>, Vincent Herbert<sup>b</sup>, Francisco Chinesta<sup>c</sup>

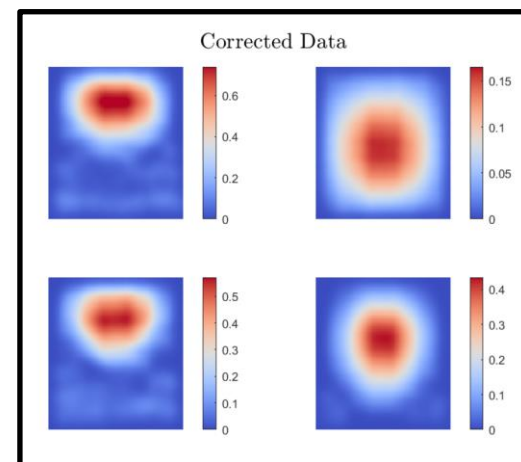
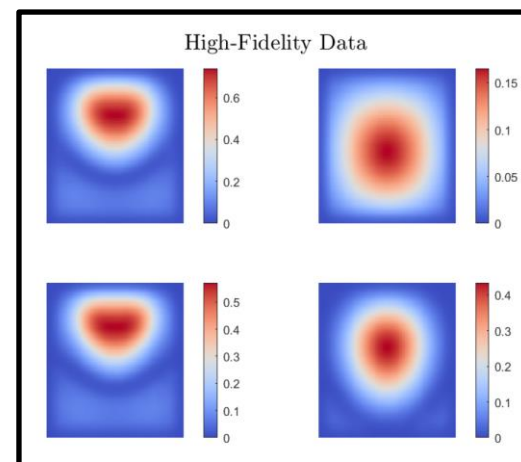
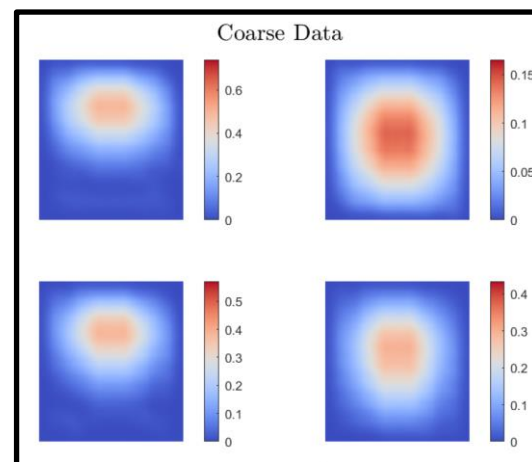
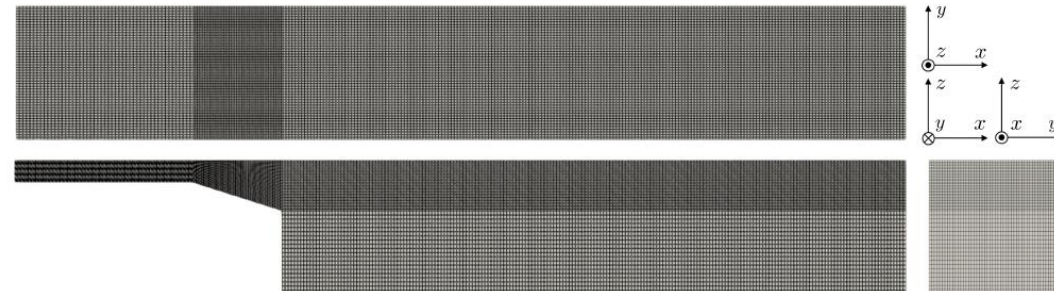
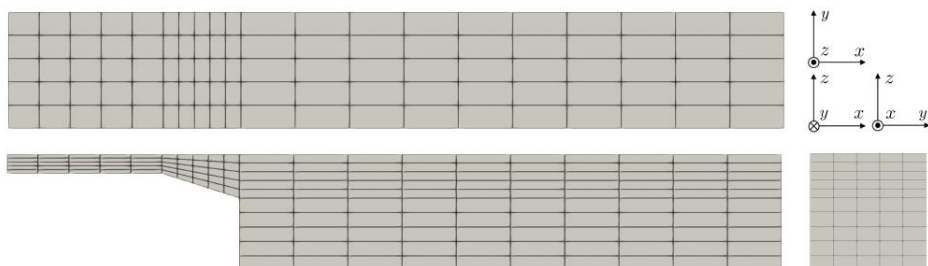
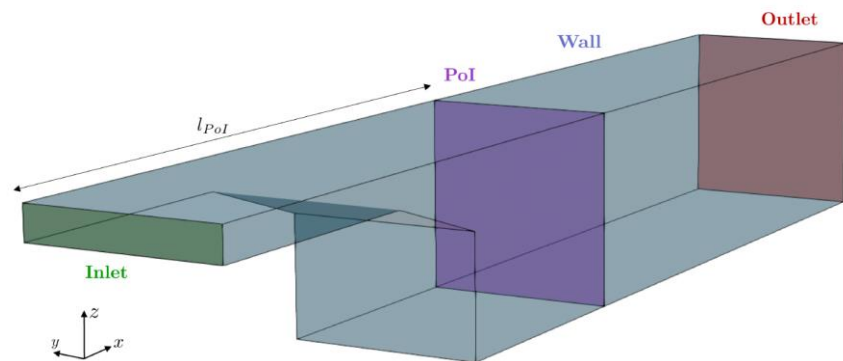
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<sup>b</sup>*STELLANTIS, Route de Gisy, 78140 Vélizy-Villacoublay, France*

<sup>c</sup>*ESI Chair, PIMM, Arts et Métiers Institute of Technology, 151 Boulevard de l'Hopital, F-75013 Paris, France*

<sup>d</sup>*ESI Chair, LAMPA, Arts et Métiers Institute of Technology, 2 Boulevard du Ronceray BP 93525, 49035 Angers cedex 01, France*





Hamiltonian formalism + QUBO  
(quadratic unconstrained binary optimization)  
**QUANTUM ANEALING**

$$\sum_{i,j,k,l}^N \sum_{p,q>p}^P X_{ij}^p \mathcal{D}(P(X_{ij}^p), P(X_{kl}^q)) X_{kl}^q$$

|                   | $\varepsilon_{max}$ [%] | $\varepsilon_{pos}$ [cm] | $W_2^2$ | $\mathcal{I}$ [%] |
|-------------------|-------------------------|--------------------------|---------|-------------------|
| Coarse Data       | 28.5                    | 2.7                      | 0.017   | 18.7              |
| HT Corrected Data | 4.1                     | 1.8                      | 0.007   | 0.8               |

DesCartes

MajuLab



